重夸克偶素产生机制：现状及未来

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Outline

I. Introduction of quarkonium

II. CSM and CEM

III. NRQCD factorization theory

IV. Soft gluon factorization

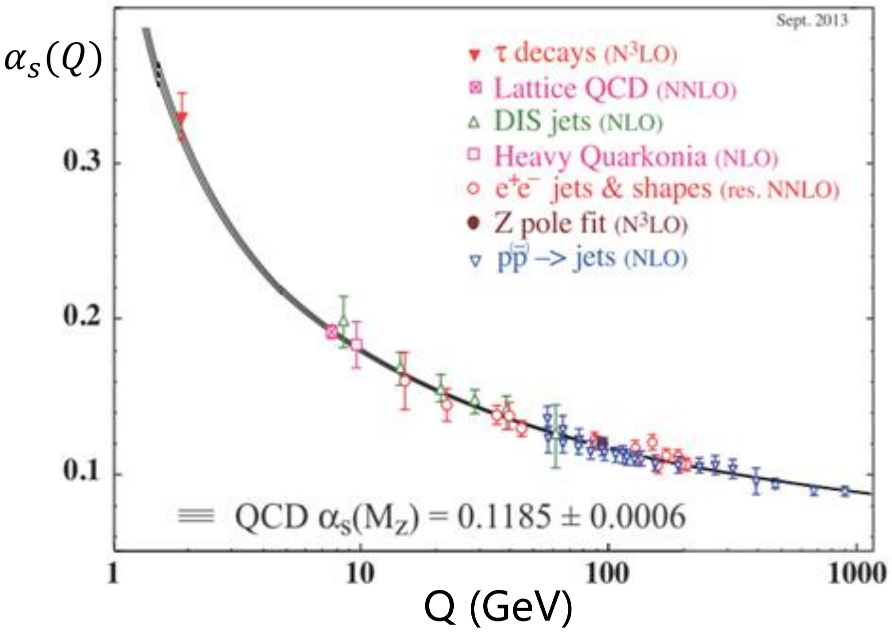
V. Recent progresses and difficulties

VI. Summary

Strong interaction and QCD

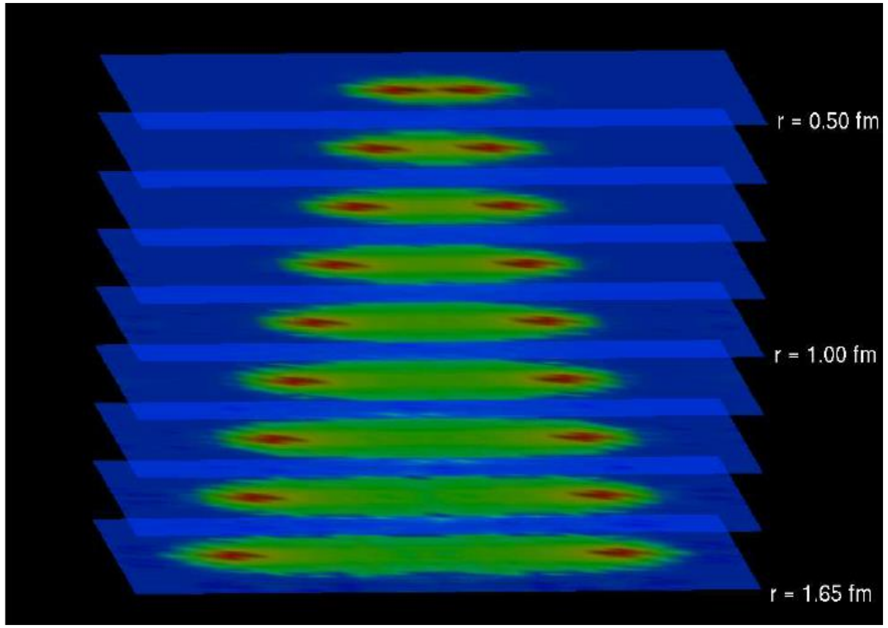
➢ Strong interaction

- Atomic bomb
- 95% mass of visible matter
- …



➢ QCD: extremely hard

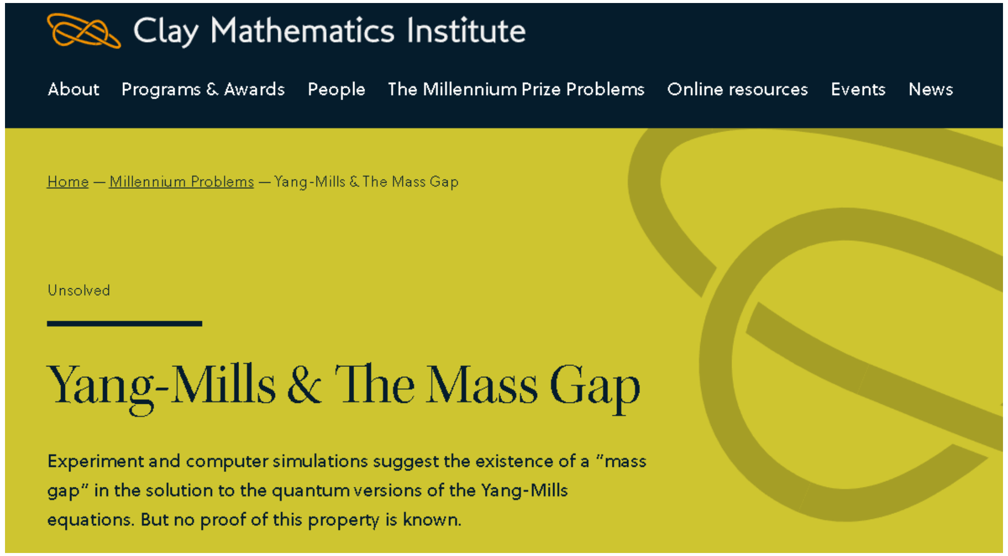
- Asymptotic freedom: perturbative at short distance
- Confinement: nonperturbative at long distance



Confinement and hadronization

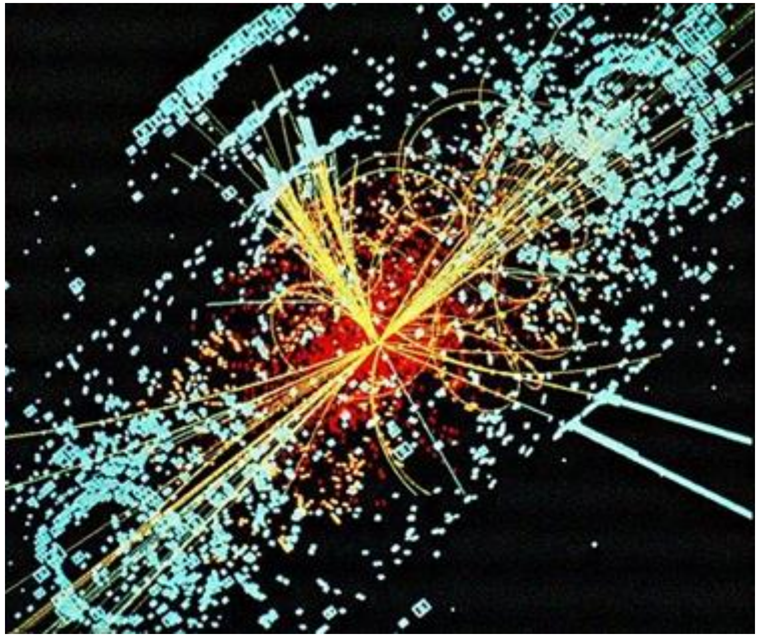
➢ Confinement

- 1/7 millennium prize problems in 21st century
- Not yet understood
- Equivalent: why and how produced quarks and gluons become hadrons?



➢ Hadronization

- Light hadrons: factorization → fragmentations functions, do not know how to compute
- Heavy quarkonium: localized color charge, perturbative QCD can help, the simplest system
- HQ production: near 50 years after the discovery, still not understood



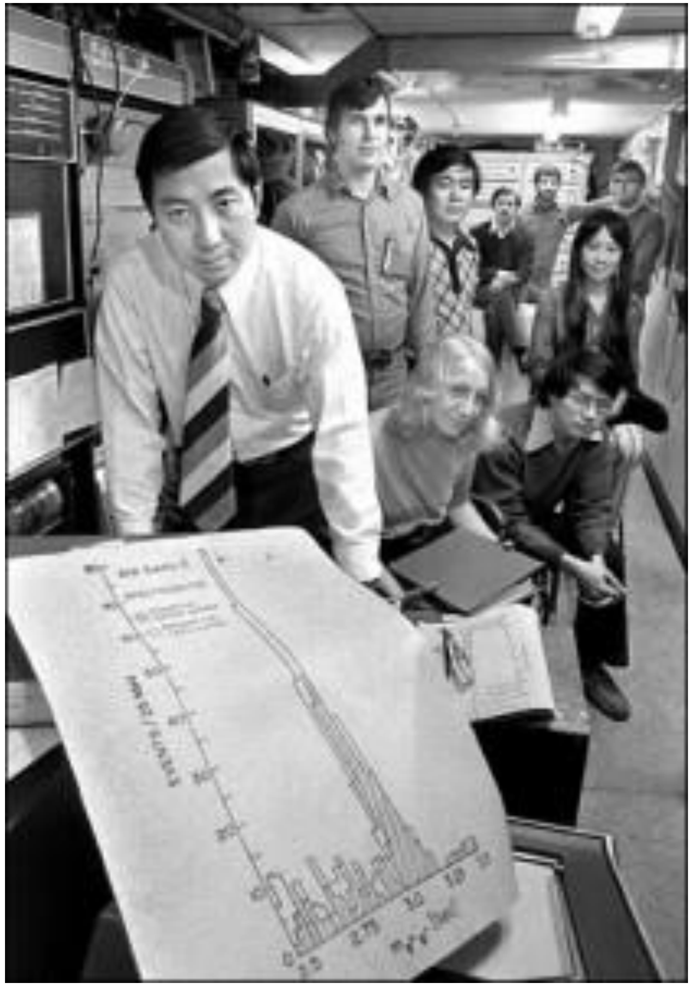
Discovery of the J/ψ: J particle

➢ GIM mechanism and charm quark

- To suppress FCNC process, Glashow–Iliopoulos–Maiani mechanism required the existence of a fourth quark

➢ J particle discovered at BNL

- In $p + Be \rightarrow e^{+} + e^{-} + X$
- 3.1 GeV, about three times heavier than the proton
- With $J^{PC} = 1^{--}$

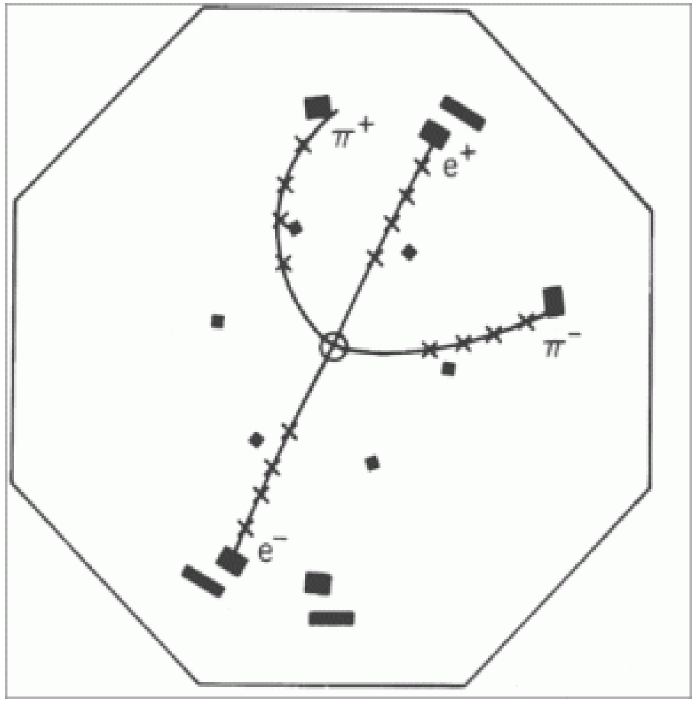


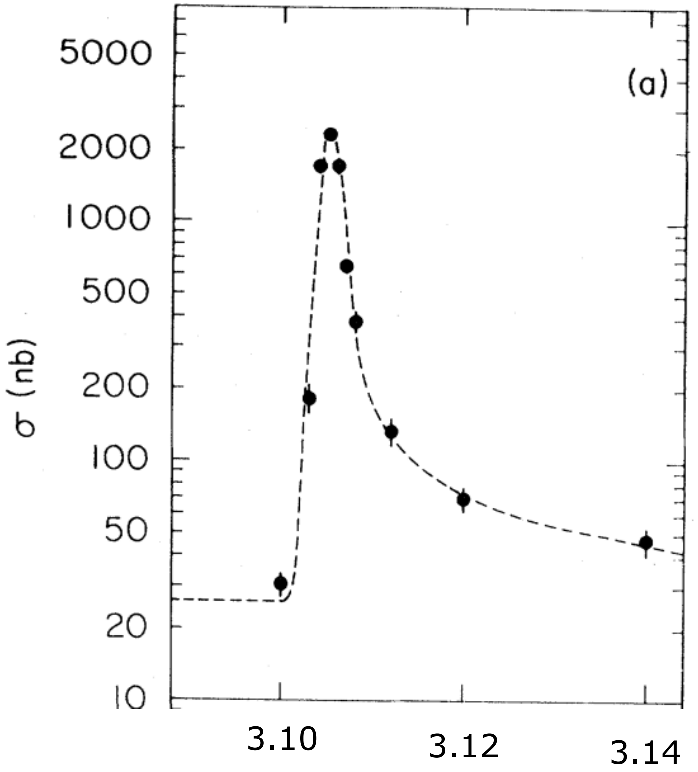
Samuel Ting and his BNL team.
Nobel Prize in 1976

Discovery of the J/ψ: ψ particle

➢ ψ particle discovered at SLAC

- In $e^+ + e^- \rightarrow \text{hadrons}, l^+l^-$



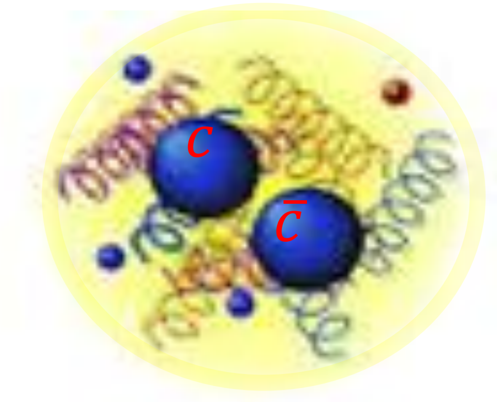


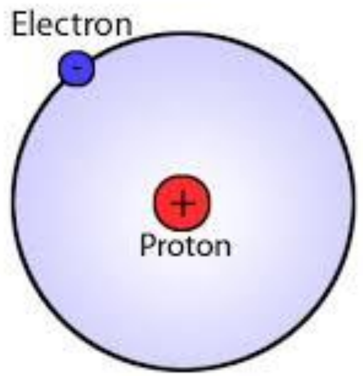
Burton Richter following the announcement of co-winning the 1976 Nobel Prize.

Heavy quarkonium

- **Bound state of $Q\bar{Q}$ pair under strong interaction**

$$J/\psi\ \ \psi', \chi_{cJ}, \Upsilon(nS), \chi_{bJ}(nP)\cdots$$





- **The simplest system in QCD: two-body problem**
- **"Hydrogen atom in QCD", "an ideal laboratory in QCD"**

Velocity of heavy quarks

- **Coulomb potential between color singlet heavy quark pair:**

$$V(r) = -C_F \frac{\alpha_s(1/r)}{r}$$

- **Virial theorem:**

$$mv^2 \sim V(r) \sim \frac{\alpha_s(1/r)}{r}$$

- **Uncertainty principle:**

$$r \sim \frac{1}{mv}$$

- **Velocity is determined by quark mass**

$$\alpha_s(mv) \sim mv^2\, r \sim v$$

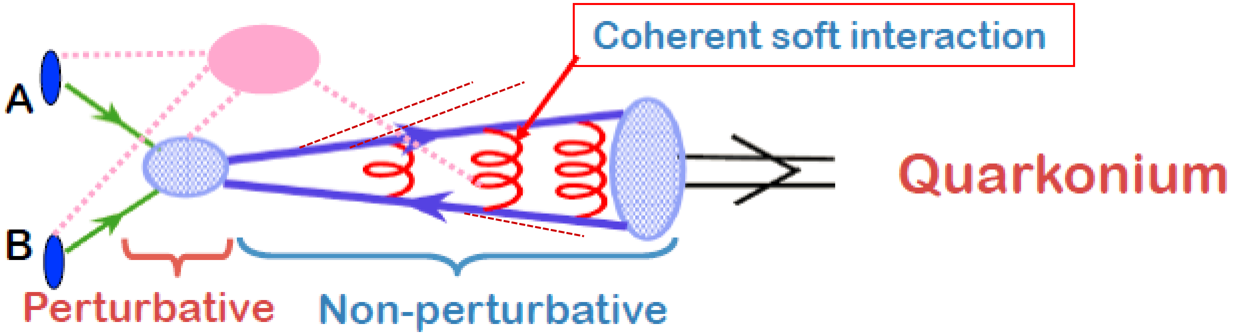
Properties

- A non-relativistic QCD system: $v^2 \ll 1$
  - Charmonium: $\mathrm{m}{\sim}1.3\mathrm{Ge}V,\ v^2 \approx 0.3$
  - Bottomonium: $m{\sim}4.5GeV,\ v^2 \approx 0.1$
- Multiple well-separated scales :
  - Quark mass: M
  - Momentum: $\mathrm{M}v$
  - Energy: Mv^2

  $\mathrm{M} \gg \mathrm{Mv} \gg \mathrm{Mv}^2 {\sim} \boldsymbol{\Lambda}_{\mathbf{QCD}}$
- Involving both perturbative and nonperturbative physics
- Production is ideal to understand hadronization: why and how quarks become hadrons?

Space-time picture for production

- Hadronization followed by production of an off-shell heavy quark pair



$$\Delta r \sim \frac{1}{2m}, \quad \frac{1}{m\,v}, \quad \frac{1}{m\,v^2}$$

- Time scale for producing heavy quark pair: $\frac{1}{2\,m}$
- Time scale for expansion: $\frac{1}{m\,v}$
- Time scale for forming bound state: $\frac{1}{m\,v^2}$

Approximation

➢ **On-shell pair + hadronization**

$$\sigma_{AB\to H+X} = \sum_n \int_n d\Gamma_{(Q\bar{Q})_n} \left[\frac{d\hat{\sigma}(Q^2)}{d\Gamma_{(Q\bar{Q})_n}}\right] F_{(Q\bar{Q})_n\to H}(p_Q, p_{\bar{Q}}, P_H)$$

- **Needs justification**
- **Corrections are at higher order in v, may need to consider**
- **Different assumptions/treatments on how the heavy quark pair becomes a heavy quarkonium: different factorization methods**

Theories for $p_T \sim m$

1. 1975 – CSM&CEM

- CSM: IR div., ψ' surplus — Einhorn, Ellis (1975), Chang (1980) …
- CEM: wrong for ratio — Fritzsch (1977), Halzen (1977) …

  (Improved-CEM: helpful in some problems) — YQM, Vogt, 1609.06042

2. 1994 - NRQCD

Bodwin, Braaten, Lepage, 9407339

- Polarization puzzle; Universality problem; Hierarchy problem; Negative cross sections; …

3. 2017- SGF

YQM, Chao, 1703.08402
Chen, YQM, 2005.08786

- Resum kinematic effects in NRQCD
- Under development; May resolve some problems

Theories for extreme p_T

➢ **High $p_T \gg m$: CO factorization**

Kang, Qiu, Sterman, 1109.1520
Fleming, Leibovich, Mehen, Rothstein 1207.2578
Kang, YQM, Qiu, Sterman, 1401.0923

- Power expansion, double parton fragmentation
- Resum large log $\ln(p_T/m)$

➢ **Low $p_T \ll m$: k_T-dependent factorization (CGC+)**

YQM, Venugopalan, 1408.4075
Watanabe, Xiao, 1507.06564

- Color Glass Condensate: resum small-x log $\ln(x)$, higher twist contributions
- Resum Sudakov log $\ln(p_T/m)$

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Color evaporation model

➢ Formula: Fritzsch (1977), Halzen (1977) …

- A fixed fraction to become ψ if the invariant mass of $c\bar{c}$-pair is below the D-meson threshold

$$\frac{d\sigma_\psi(P)}{d^3P} = F_\psi \int_{2m_c}^{2M_D} dM \frac{d\sigma_{c\bar{c}}(M,P)}{dM\,d^3P}$$

Simple and intuitive

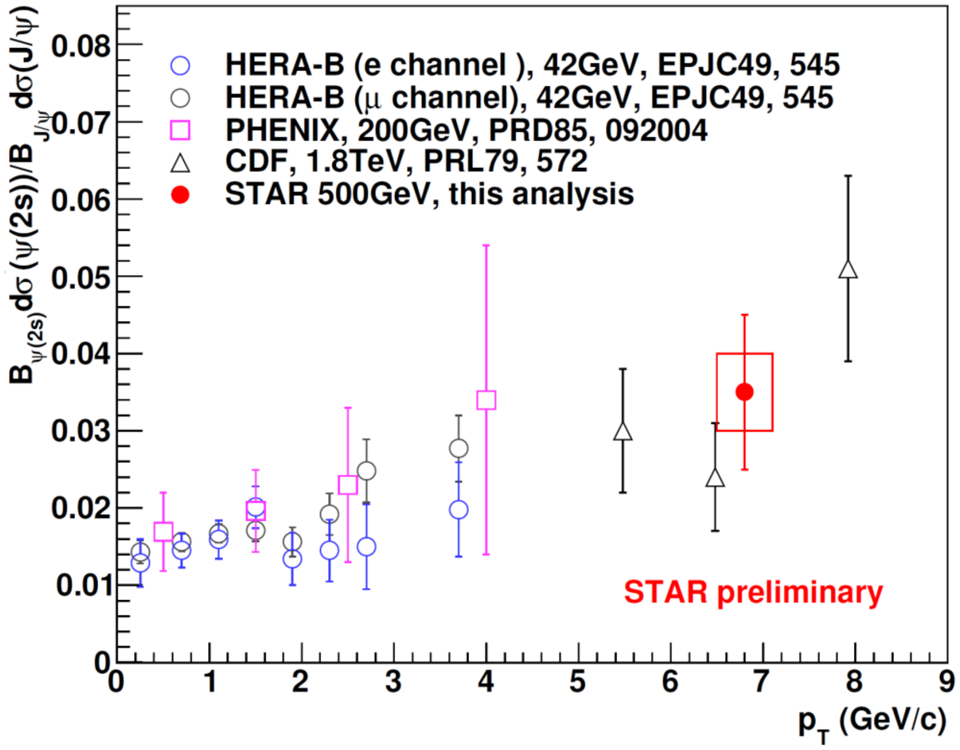
One parameter for each quarkonium

No IR div. problem

CEM - wrong for ratio

➢ But

- Wrong prediction for ratio of two quarkonia: constant
- Data: kinematics dependent



Improved CEM

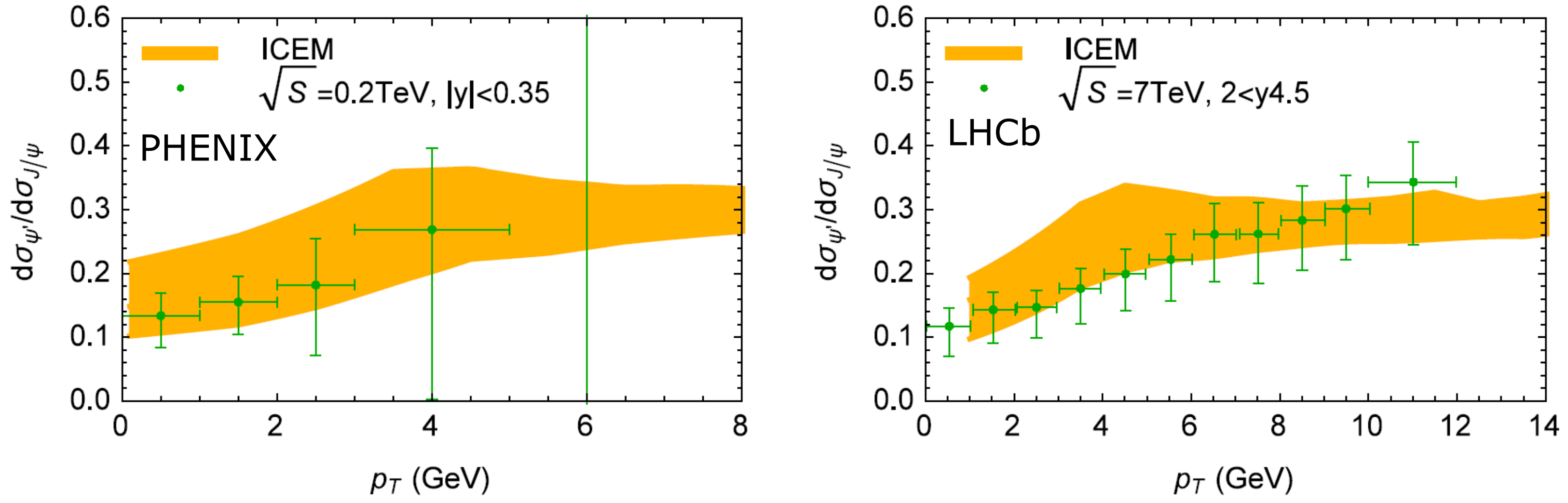
- **The improved model:**

YQM, Vogt, 1609.06042

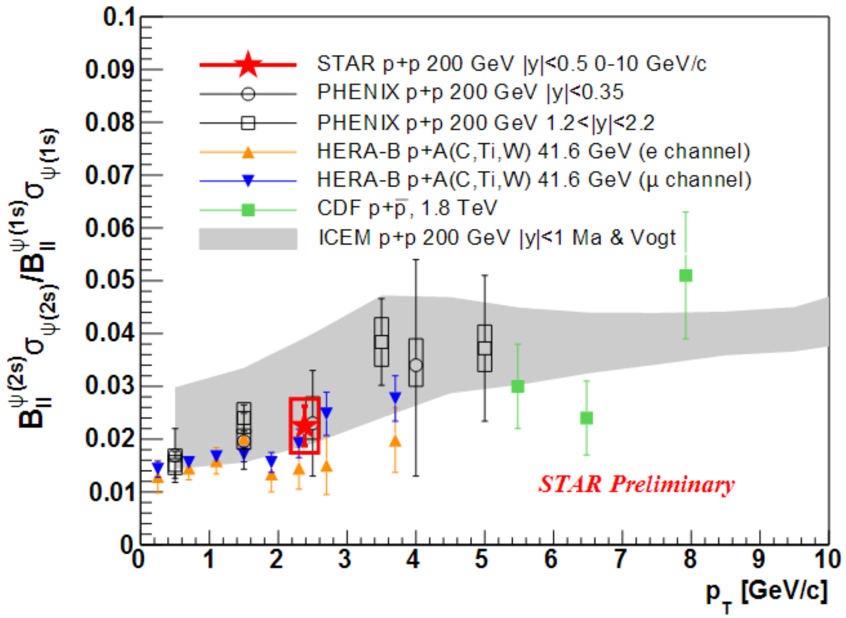
$$\frac{d\sigma_\psi(P)}{d^3P} = F_\psi \int_{M_\psi}^{2M_D} d^3P' dM \frac{d\sigma_{c\bar{c}}(M,P')}{dMd^3P'} \delta^3\left(P - \frac{M_\psi}{M}P'\right)$$

- **Momentum shift due to soft particles emission in hadronizaiton process**
- **Comparing with traditional CEM:**

$$\frac{d\sigma_\psi(P)}{d^3P} = F_\psi \int_{2m_c}^{2M_D} dM \frac{d\sigma_{c\bar{c}}(M,P)}{dMd^3P}$$



Problem of CEM



- CEM ignores quantum numbers of quarkonia
- ICEM can describe production ratio $\psi(2S)/\psi(1S)$, which has similar quantum numbers
- Still hard to describe other ratios, like $\chi_{cJ}/\psi(1S)$

Color-singlet model

➢ CSM:

Einhorn, Ellis (1975), Chang (1980) …

- Production cross section of a $c\bar{c}$-pair with zero relative momentum and same quantum numbers as that of ψ, multiplied by wave function at origin
- Simply and intuitive
- Effective no free parameter (potential model)

➢ Problems

- Theoretically: IR divergence at NLO level for P-wave quarkonium
- Phenomenologically: fine until 1990s

CSM – ψ' puzzle

➢ **CDF: a surprisingly large production rate**

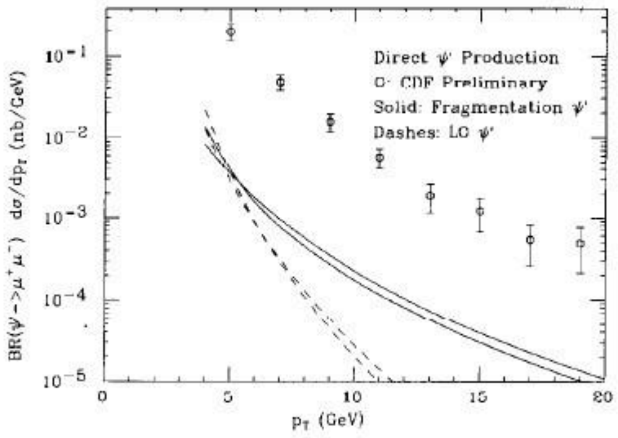


Fig. 4. Preliminary CDF data for prompt ψ' production (O) compared with theoretical predictions of the total fragmentation contribution (solid curves) and the total leading-order contribution (dashed curves).

Braaten, Doncheski, Fleming, Mangano (1994)

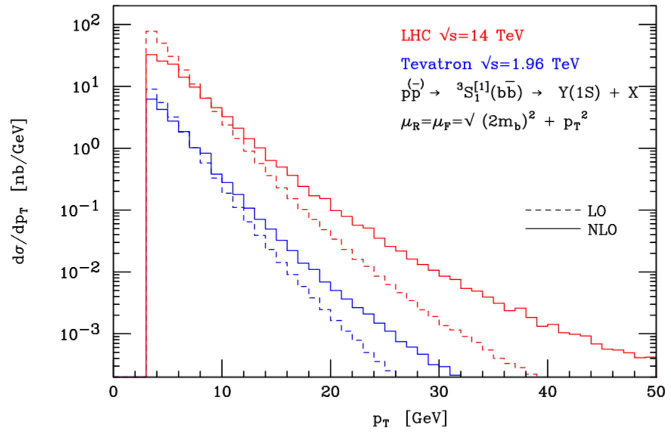
- **Larger than the CSM prediction by a factor of 30, even though including the fragmentation contribution**

CSM – NLO and NNLO* calculation

➢ Complete NLO calculation

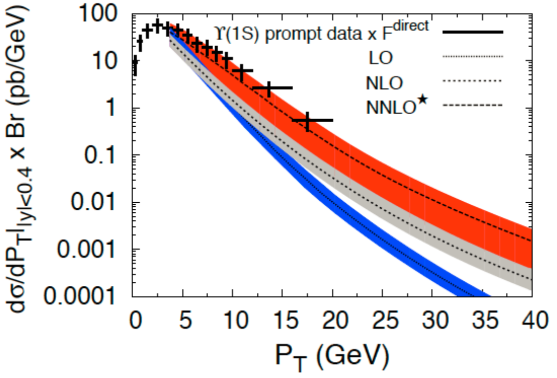
Campbell, Maltoni, Tramontano, 0703113
Gong, Wang, 0802.3727

- Larger than LO by orders
- Still much smaller than data

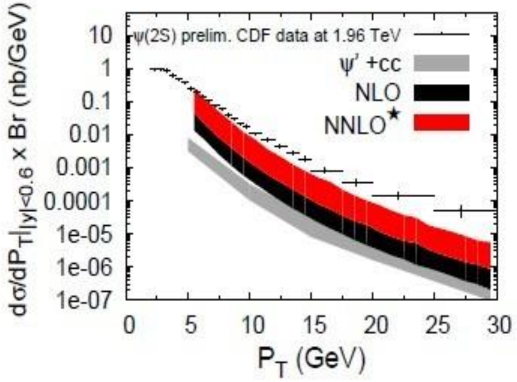


➢ NNLO* : Estimate NNLO by tree level diagrams

- Almost reach the data
- Infrared cutoff s_{ij}^{min} denpendent



Artoisenet, Campbell, Lansberg, Maltoni, Tramontano, 0806.3282



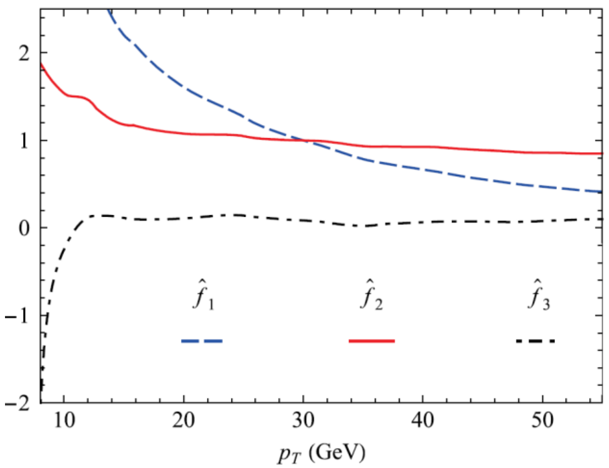
Lansberg, 0811.4005

CSM – Problem with NNLO*

➢ **Cannot explain experiment data by including NNLO contribution**

YQM, Wang, Chao, 1012.1030

- NNLO*: **dominated by double logarithm, which will be canceled by loop corrections**
- **Thus** NNLO* **method overestimates the CSM contribution**



$$R^\star = d\sigma_{NNLO^\star}/d\sigma_{NLO}$$

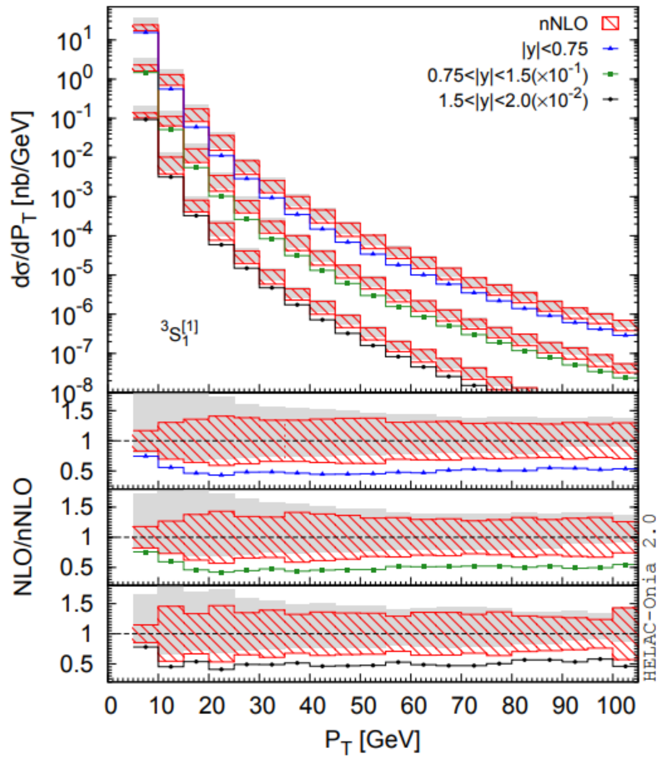
$$\hat{f}_1 = \frac{R^\star}{p_T^2}\Big|_{s_{ij}^{min}=0.5m_b^2}$$

$$\hat{f}_2 = \frac{R^\star}{\log^2(p_T^2/s_{ij}^{min})}\Big|_{s_{ij}^{min}=0.5m_b^2}$$

$$\hat{f}_3 = 1 - \frac{R^\star/\log^2(p_T^2/s_{ij}^{min})\,|_{s_{ij}^{min}=2m_b^2}}{R^\star/\log^2(p_T^2/s_{ij}^{min})\,|_{s_{ij}^{min}=0.5m_b^2}}$$

Shao, 1809.02369

- **By using an infrared-save subtraction method, nNLO correction is not that sizable**



Full NNLO is hard to explain data

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NRQCD: factorization

➢ Factorization formula

Bodwin, Braaten, Lepage, 9407339

$$(2\pi)^3 2P_H^0 \frac{d\sigma_H}{d^3P_H} = \sum_n d\hat{\sigma}_n(P_H)\langle\mathcal{O}_n^H\rangle$$

Production of a heavy quark pair
Expansion in: α_s

Hadronization (LDMEs)
Expansion in: v

- n: quantum numbers of the pair: color, spin, orbital angular momentum, total angular momentum, spectroscopic notation ${}^{2S+1}L_J^{[c]}$

➢ A glory history

- Solved IR divergences in P-wave quarkonium decay
- Explained ψ' surplus
- Explained χ_{c2}/χ_{c1} production ratio
- ....

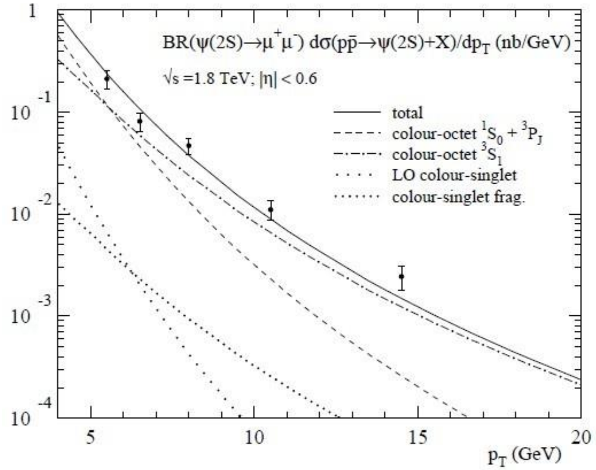
Thanks to color-octet mechanism

Examples of achievements

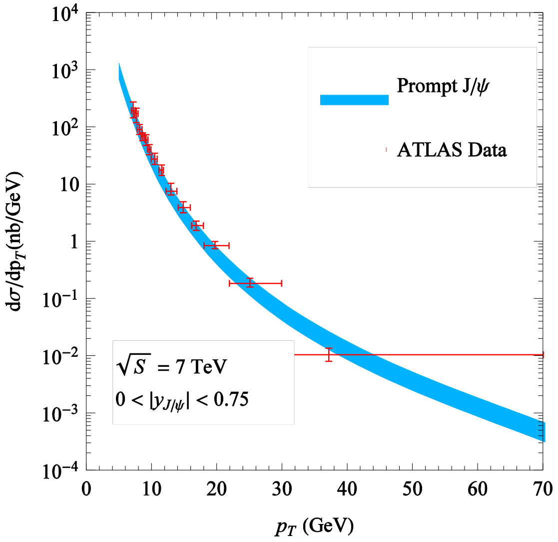
CO mechanism

- **Nicely explain ψ' surplus by CO contributions**

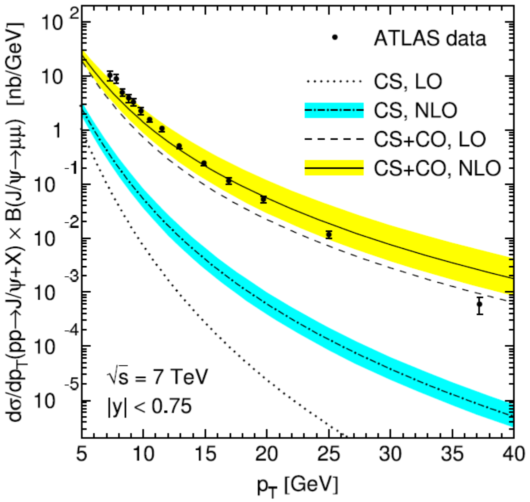
States	p_T behavior at LO
${}^3S_1^{[1]}$	p_T^{-8}
${}^3S_1^{[8]}$	p_T^{-4}
${}^1S_0^{[8]}$	p_T^{-6}
${}^3P_J^{[8]}$	p_T^{-6}



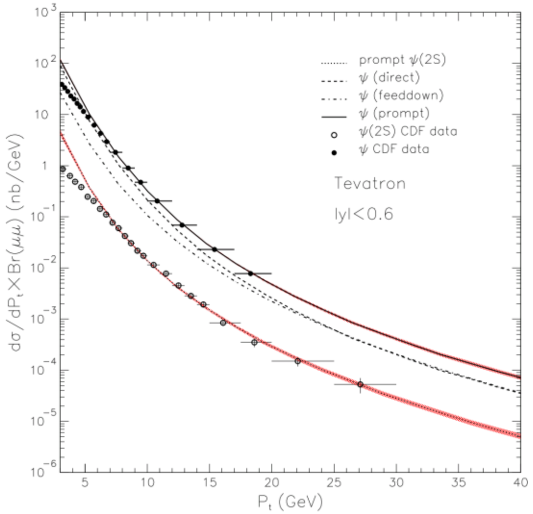
Kramer, 0106120



YQM, Wang, Chao, 1012.1030



Butenschoen, Kniehl, 1105.0820

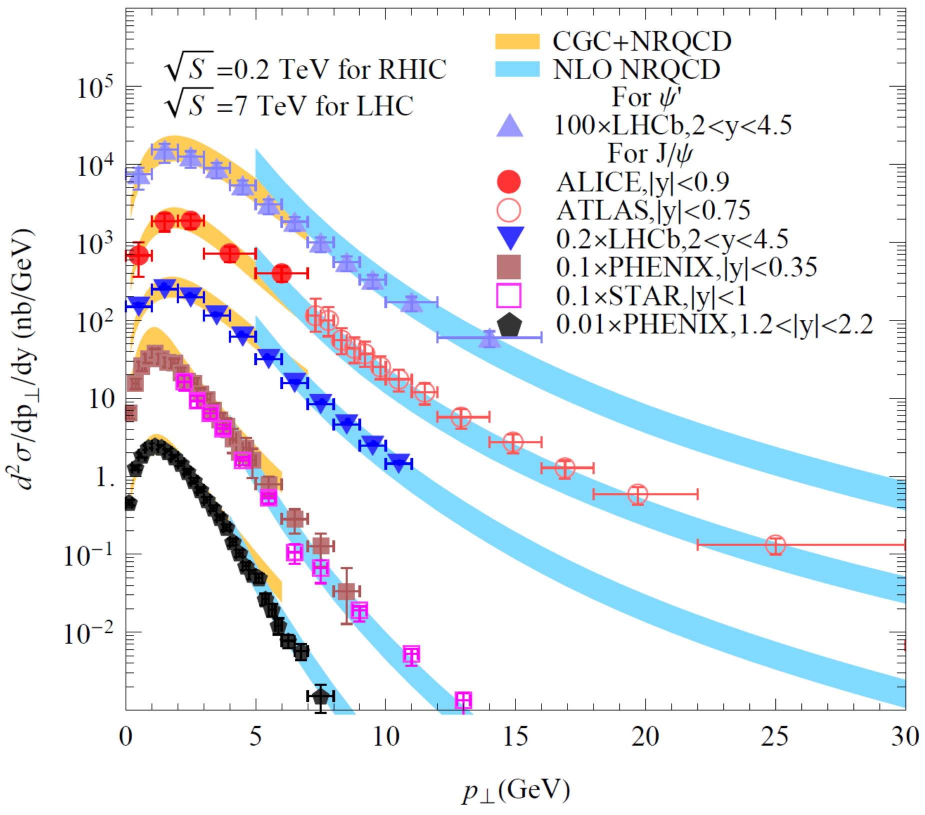


Gong, Wan, Wang, Zhang, 1205.6682

Small p_T region

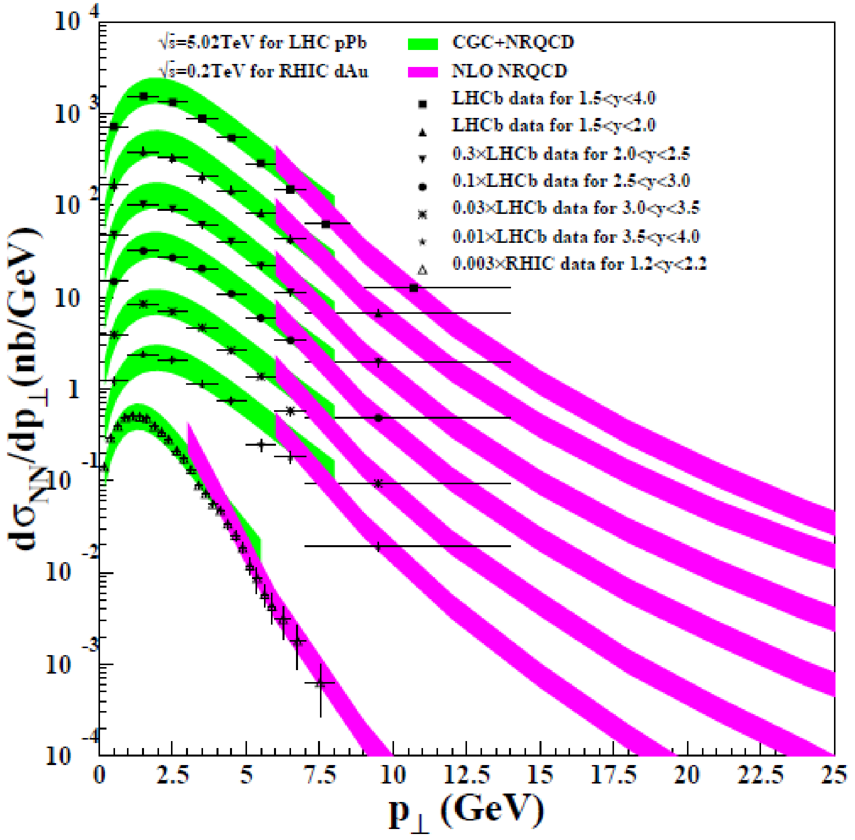
➢ CGC+NRQCD : comprehensive description of $\psi(nS)$ production

Ma, Venugopalan, 1408.4075



$\psi(nS)$ in p+p collisions

Ma, Venugopalan, Zhang, 1503.07772



J/ψ in p+A collisions

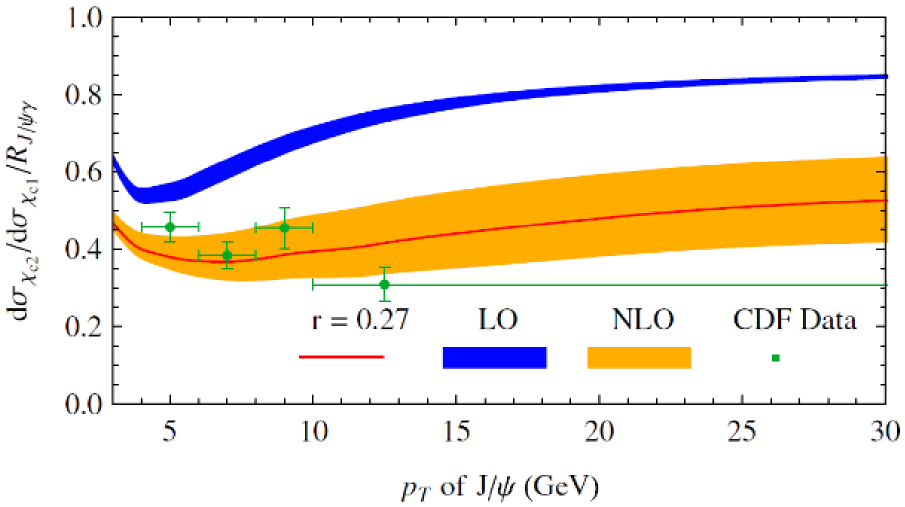
Predictions for χ_{cJ} production

- χ_{cJ} **production:** YQM, Wang, Chao, 1002.3987

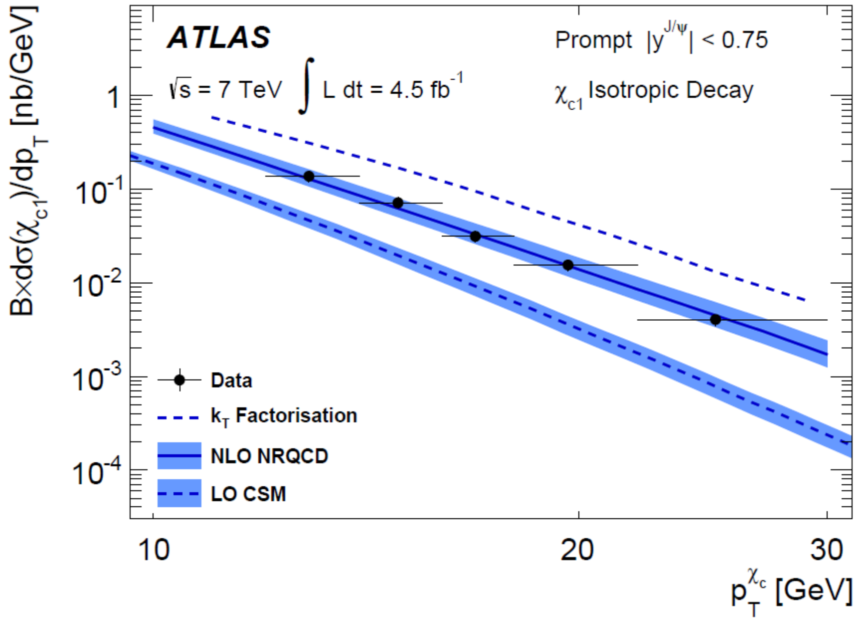
$$d\sigma_{\chi_{cJ}} \approx d\hat{\sigma}_{^3P_J^{[1]}} \langle O\left({}^3P_0^{[1]}\right)\rangle + (2J+1) d\hat{\sigma}_{^3S_1^{[8]}} \langle O\left({}^3S_1^{[8]}\right)\rangle$$

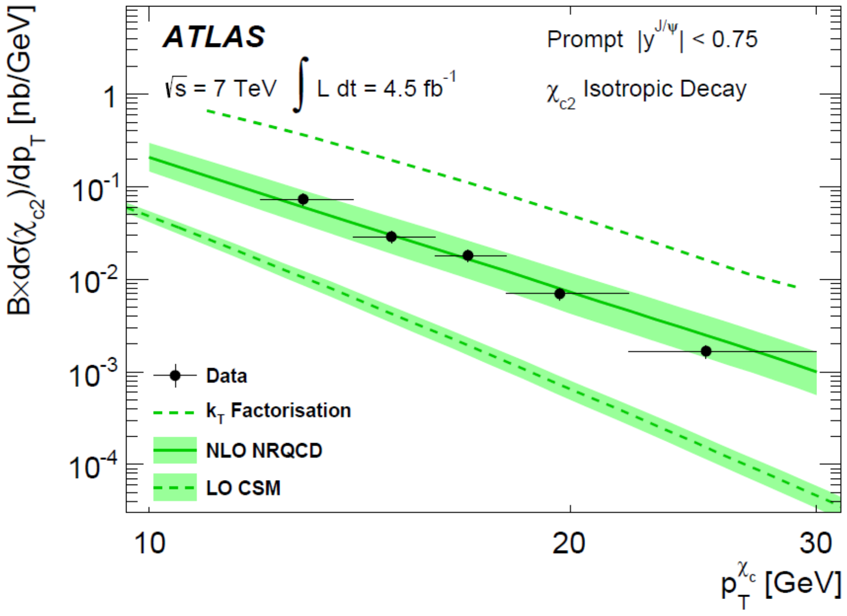
CEM predicts: $R_{\chi_c} = \sigma_{\chi_{c2}}/\sigma_{\chi_{c1}} = 5/3$

LO NRQCD at high p_T: $R_{\chi_c} = 5/3$



- **Agree with later data** ATLAS, 1404.7035





Difficulties of NRQCD

Hierarchy and universality problems

- **Fit J/ψ yield data at Tevatron with $p_T > 7$ GeV**

YQM, Wang, Chao, 1009.3655

  - Due to p_T^{-4} and p_T^{-6} behaviors, constrain two combinations
  - $M_0 = \langle O\left({}^1S_0^{[8]}\right)\rangle + 3.9\,\langle O\left({}^3P_0^{[8]}\right)\rangle/m_c^2 \approx (7.4 \pm 1.9)\times 10^{-2}\,\text{GeV}^3$
  - $M_1 = \langle O\left({}^3S_1^{[8]}\right)\rangle - 0.56\,\langle O\left({}^3P_0^{[8]}\right)\rangle/m_c^2 \approx (0.05 \pm 0.02)\times 10^{-2}\,\text{GeV}^3$

See also:
Butenschoen, Kniehl, 1105.0820
Gong, Wan, Wang, Zhang, 1205.6682

- **Two orders difference: hierarchy problem**
  - Velocity scaling rule of NRQCD

$$\langle O\left({}^1S_0^{[8]}\right)\rangle \sim \langle O\left({}^3S_1^{[8]}\right)\rangle \sim \langle O\left({}^3P_0^{[8]}\right)\rangle/m_c^2$$

  - Thus natural expectation: $M_0 \sim M_1$

- **Upper bound from Belle total cross section**

$$M_0 < 0.02\text{GeV}^3$$

Zhang, YQM, Wang, Chao, 0911.2166

  - No universality of NRQCD LDMEs!

Polarization puzzle at LO

➢ LO NRQCD

- **Dominated by $^3S_1^{[8]}$, LO NRQCD predicts transversely polarized $\psi(\mathrm{nS})$, contradicts with CDF data**

CDF, 0704.0638

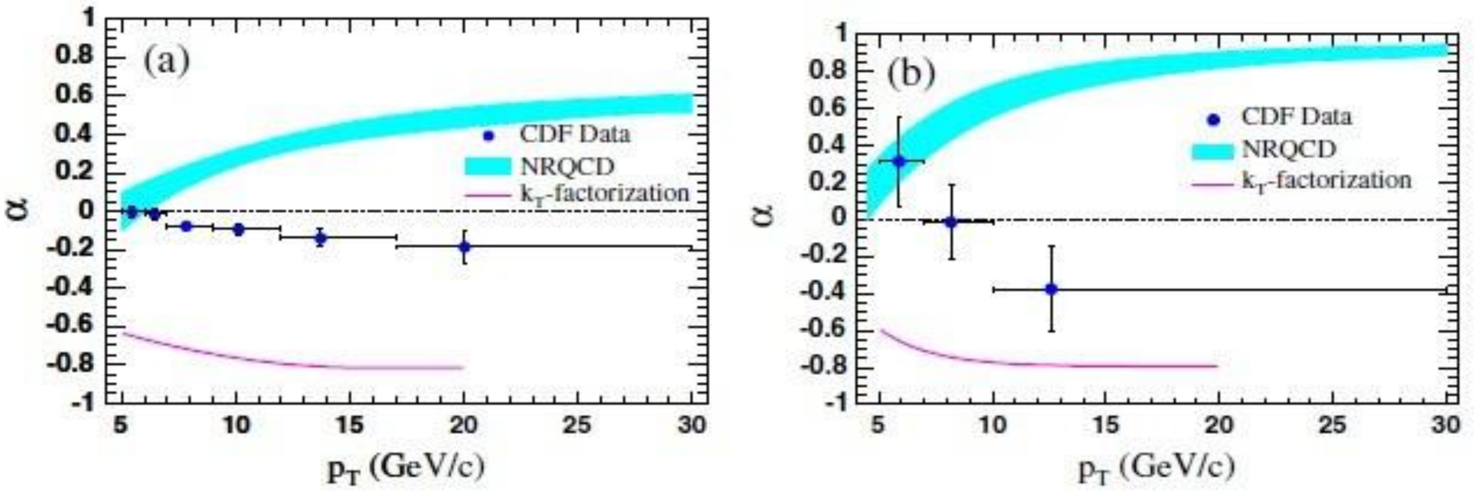
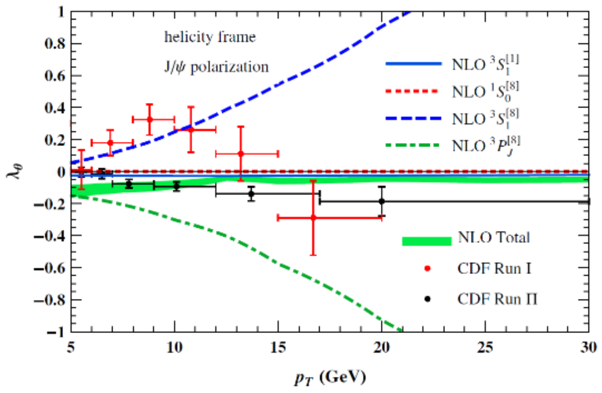


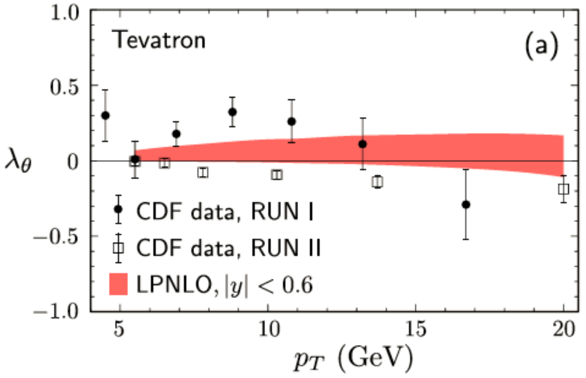
FIG. 4 (color online). Prompt polarizations as functions of p_T: (a) J/ψ and (b) $\psi(2S)$. The band (line) is the prediction from NRQCD [4] (the k_T-factorization model [9]).

Polarization puzzle at NLO

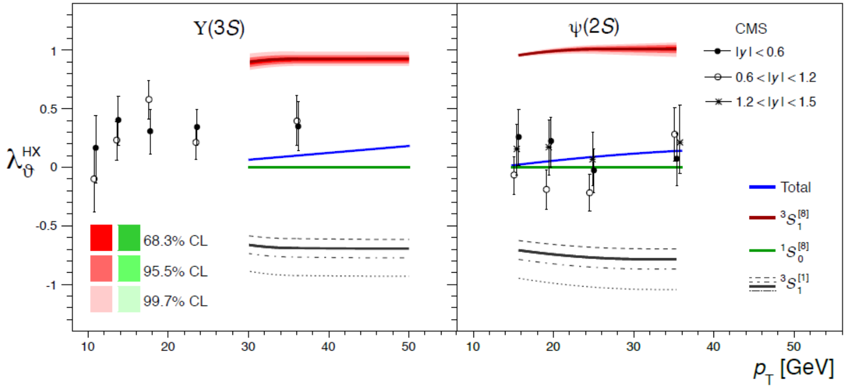
➢ J/ψ: transverse canceled (results in hierarchy) in ${}^3S_1^{[8]}$ and ${}^3P_J^{[8]}$



Chao,YQM,Shao,Wang, Zhang,1201.2675

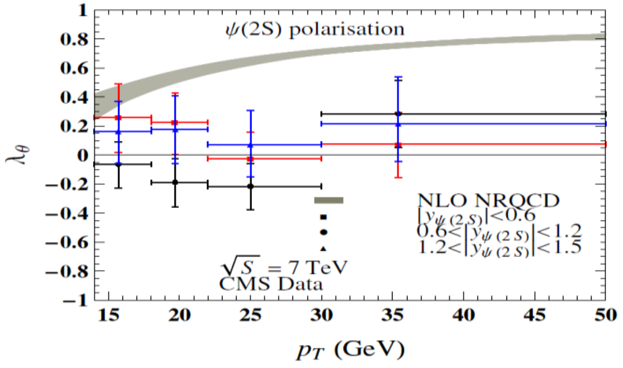


Bodwin, Chung, Kim, Lee, 1403.3612

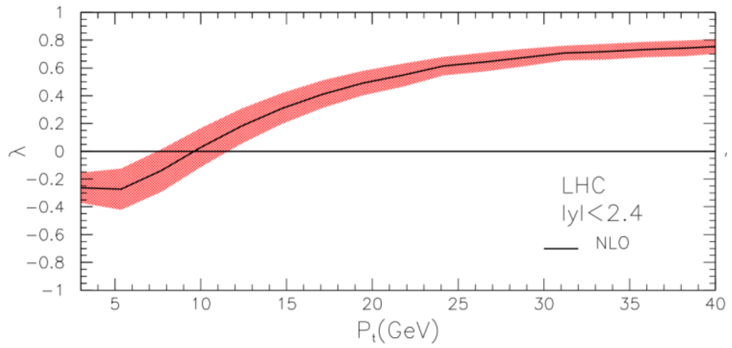


Faccioli,Knunz,Lourenco, Seixas,Wohri,1403.3970

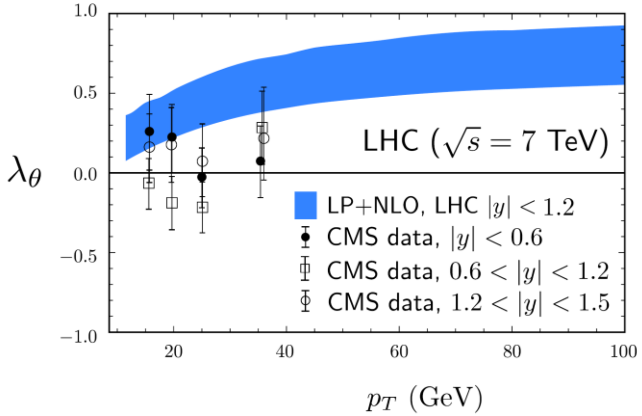
➢ $\psi(2S)$: cancelation weak, **hard to understand data**



Shao, Han, YQM, Meng, Zhang, Chao, 1411.3300



Gong, Wan, Wang, Zhang, 1205.6682



Bodwin et al., 1509.07904

η_c production: spin symmetry broken?

➢ **Heavy quark spin symmetry (HQSS)**

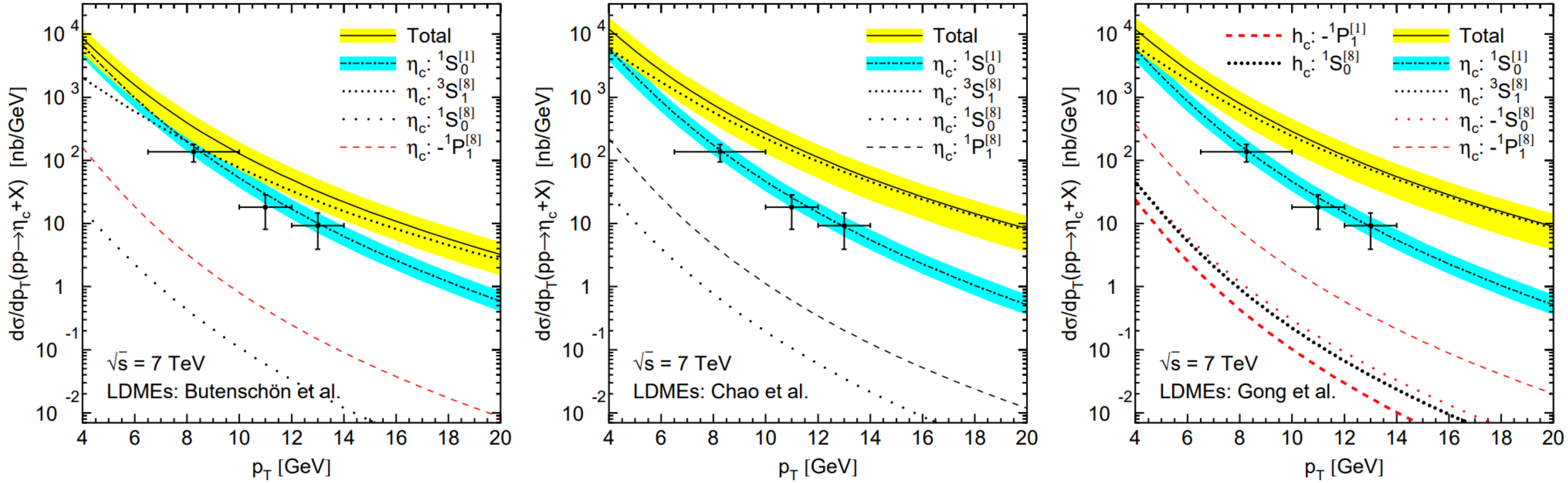
$$\left\langle \mathcal{O}^{\eta_c}\left({}^3S_1^{[8]}\right)\right\rangle = \left\langle \mathcal{O}^{J/\psi}\left({}^1S_0^{[8]}\right)\right\rangle,$$

$$\left\langle \mathcal{O}^{\eta_c}\left({}^1S_0^{[8]}\right)\right\rangle = \frac{1}{3}\left\langle \mathcal{O}^{J/\psi}\left({}^3S_1^{[8]}\right)\right\rangle,$$

$$\left\langle \mathcal{O}^{\eta_c}\left({}^1P_1^{[8]}\right)\right\rangle = \frac{3}{2J+1}\left\langle \mathcal{O}^{J/\psi}\left({}^3P_J^{[8]}\right)\right\rangle.$$

- Using the J/ψ LDMEs extracted by various groups, NLO NRQCD predictions greatly overshoot the LHCb data

Butenschoen, He, Kniehl,1411.5287



- Possible solutions: large cancelation between S-wave and P-wave (results in hierarchy)

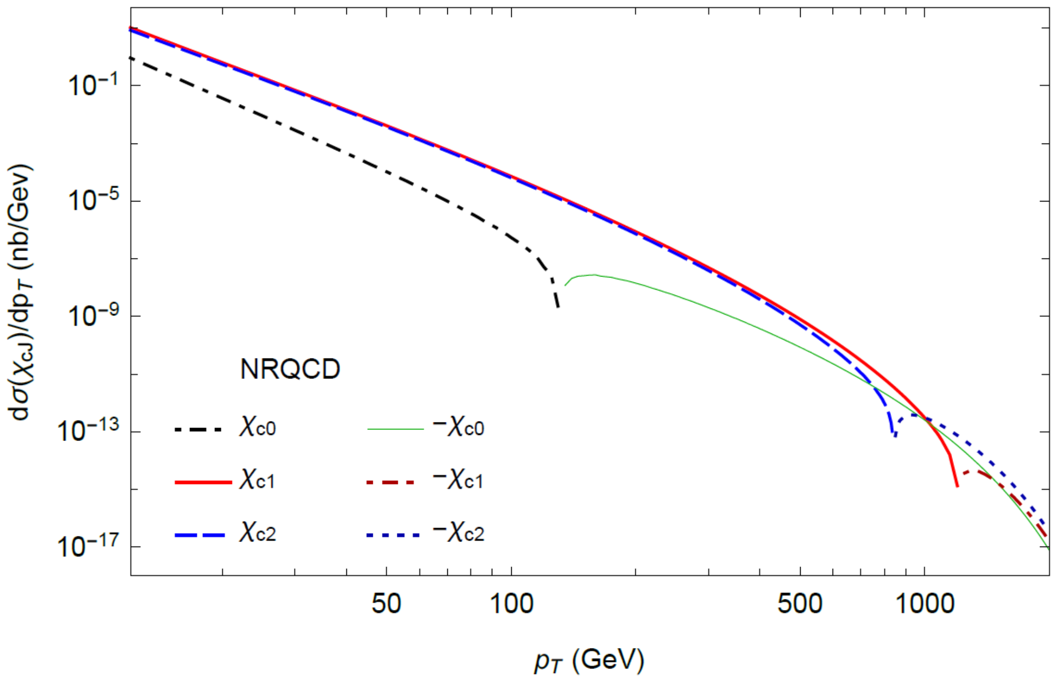
Han, YQM, Meng, Shao, Chao, 1411.7350
Zhang, Sun, Sang, Li, 1412.0508

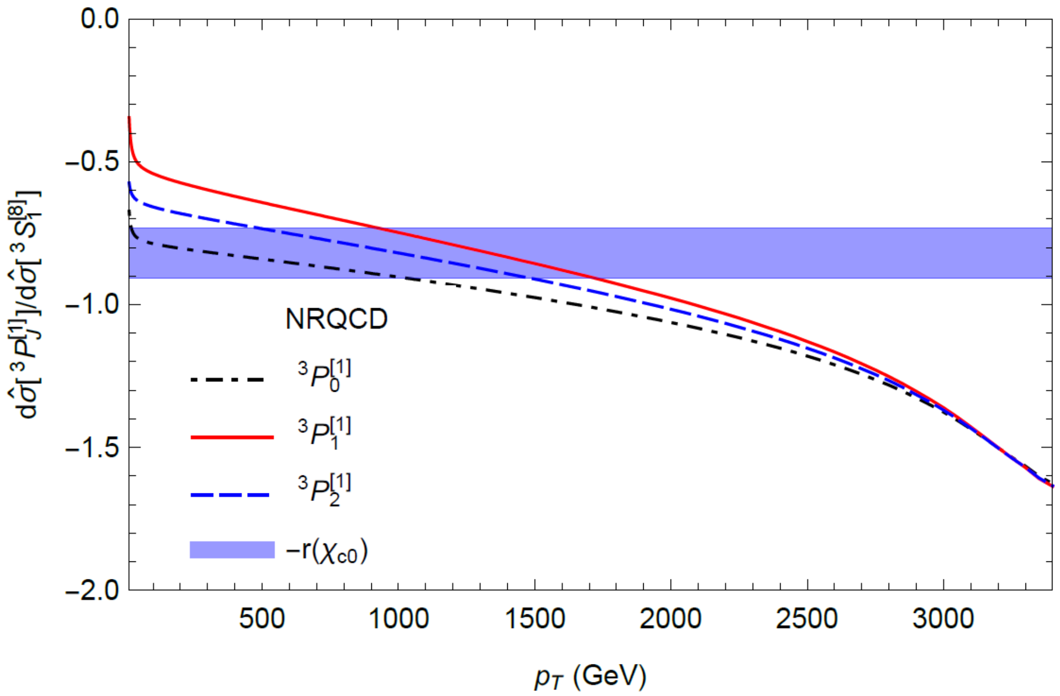
Negative differential cross sections

➢ **Cross sections become negative at exceptionally high p_T**

$$d\sigma(\chi_{cJ}) = (2J+1)d\hat{\sigma}[^3S_1^{[8]}]\frac{\langle\mathcal{O}^{\chi_{c0}}(^3P_0^{[1]})\rangle}{m_c^2} \times\left[r(\chi_{c0}) + \frac{d\hat{\sigma}[^3P_J^{[1]}]}{d\hat{\sigma}[^3S_1^{[8]}]}\right].$$

$$r(\chi_{c0}) \equiv \frac{\langle\mathcal{O}^{\chi_{c0}}(^3S_1^{[8]})\rangle}{\langle\mathcal{O}^{\chi_{c0}}(^3P_0^{[1]})\rangle/m_c^2},$$





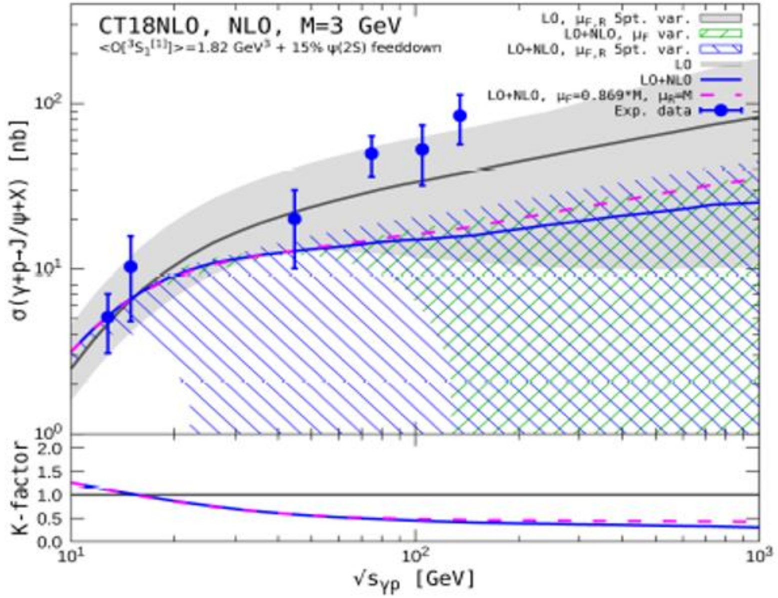
Negative total cross sections

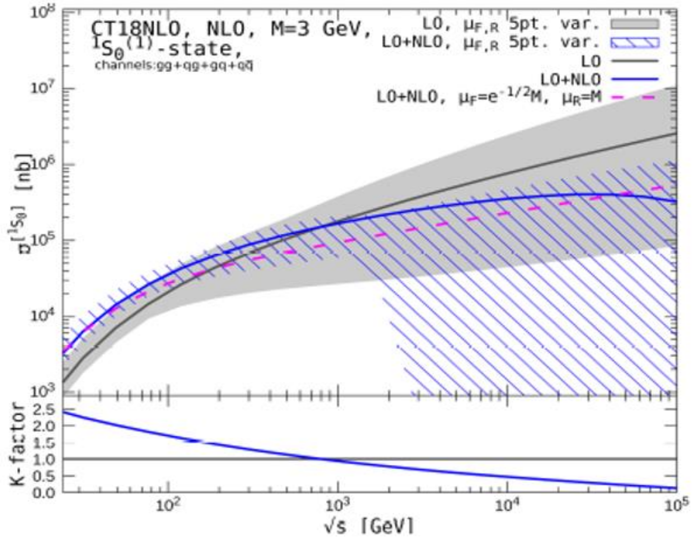
Inclusive J/ψ-photoproduction (CSM):

$$\gamma + p \to c\bar{c}\left[{}^3S_1^{[1]}\right] + X, \ \text{LO: } \gamma(q) + g(p_1) \to c\bar{c}\left[{}^3S_1^{[1]}\right] + g,$$

Inclusive η_c-hadroproduction (CSM):

$$p + p \to c\bar{c}\left[{}^1S_0^{[1]}\right] + X, \ \text{LO: } g(p_1) + g(p_2) \to c\bar{c}\left[{}^1S_0^{[1]}\right]$$



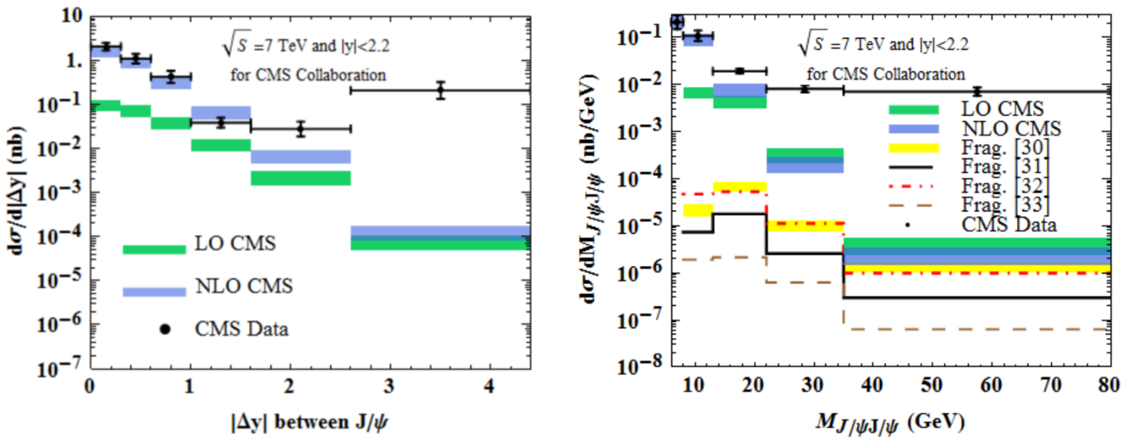


Lansberg, Nefedov, Ozcelik, 2112.06789

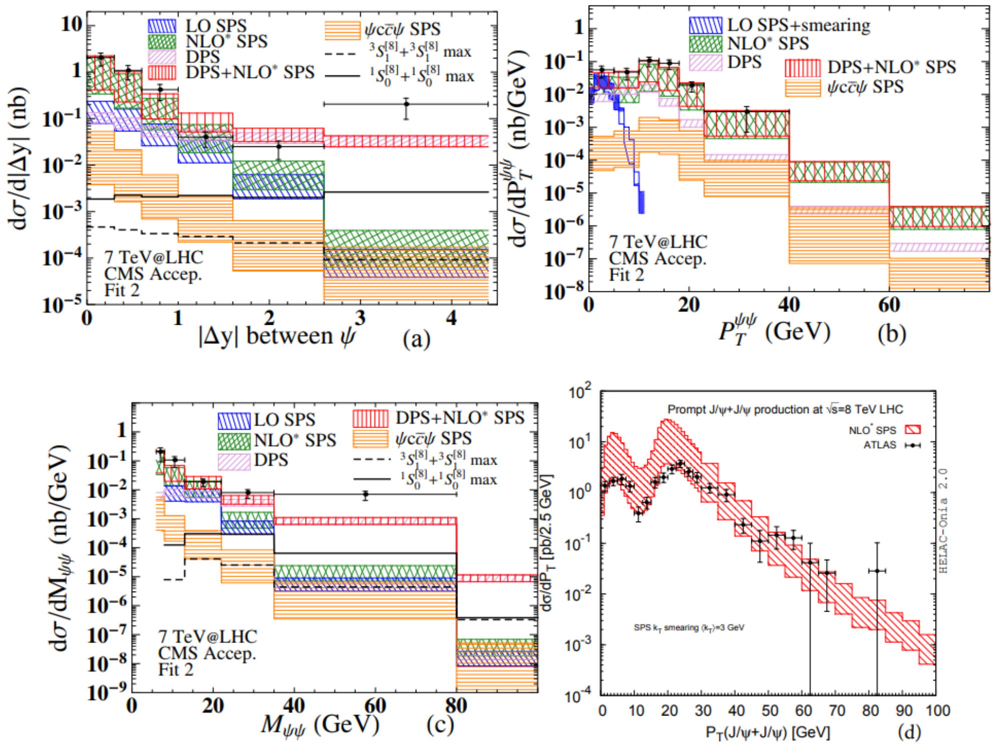
Double J/ψ production

➢ Cannot explain data

- 3 orders of discrepancy between data and single-parton scattering
- 1 order discrepancy still exist after including double-parton scattering
- What is missing?



Sun, Han, Chao, 1404.4042

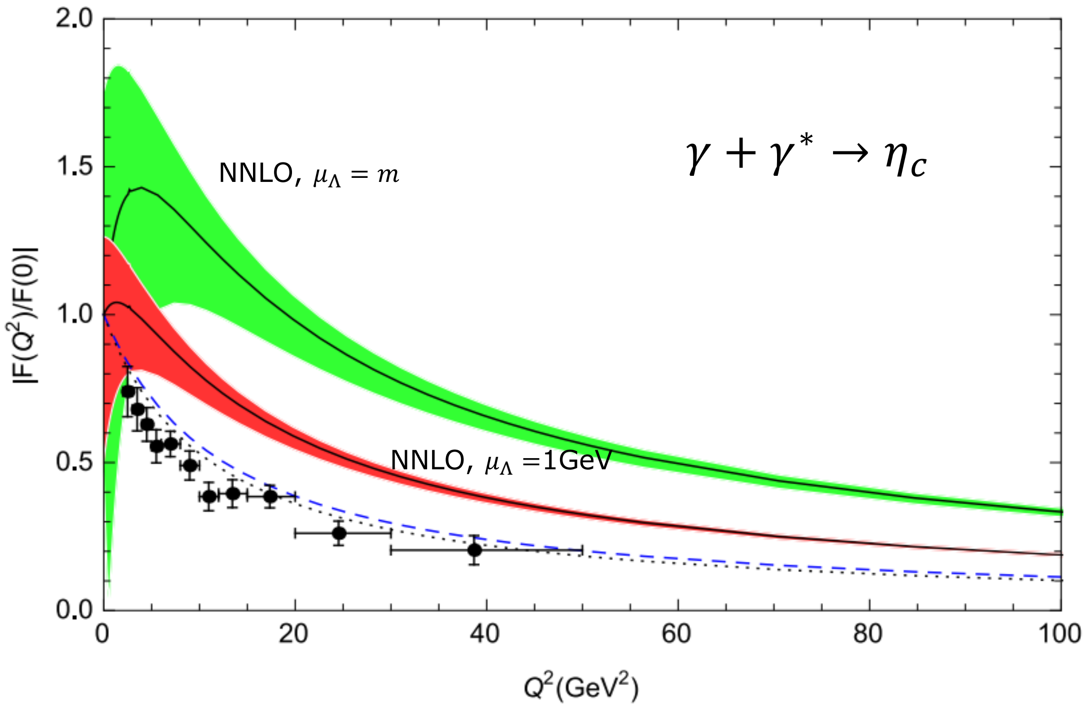


Lansberg, Shao, 1410.8822

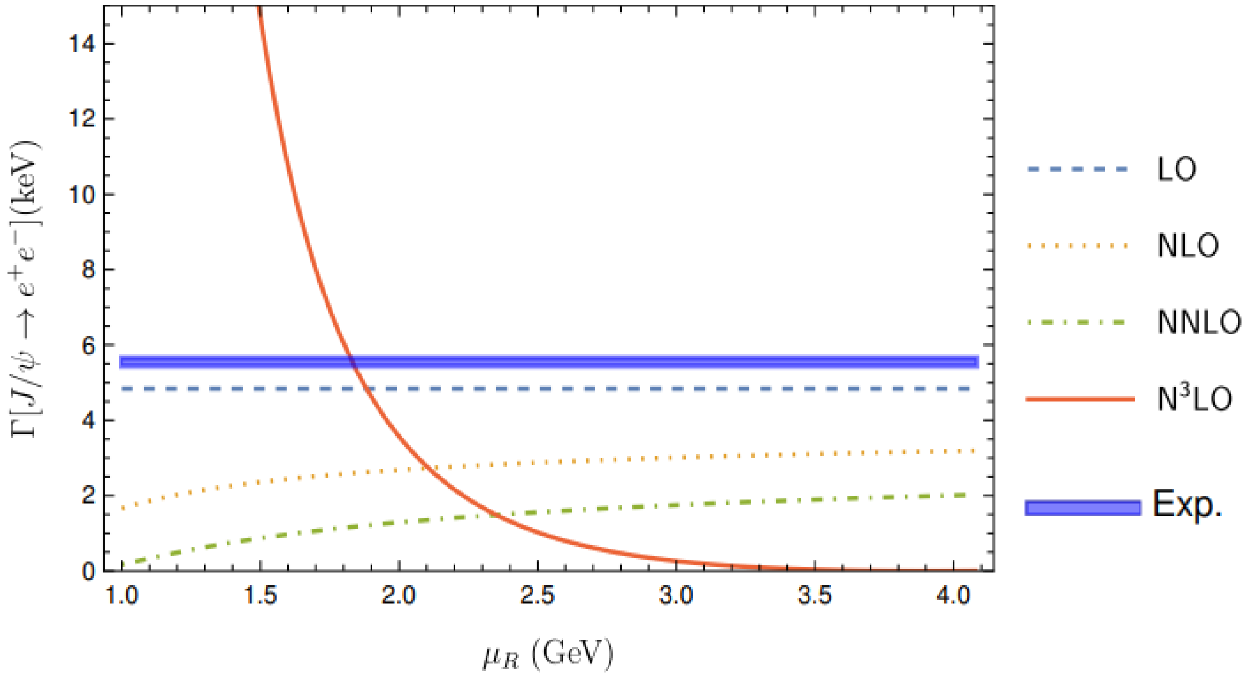
Beyond NLO

- **Very big high order correction!**
  - **Higher orders fail to describe data**
  - **Strange: breaking down of perturbation theory? Or other mechanism?**

Feng, Jia, Sang, 1505.02665



Feng, Jia, Mo, Pan, Sang, Zhang, 2207.14259



物理真南！

Status around 2015

➢ Phenomenological: difficulties

- NLO NRQCD theory has difficulty to describe global data: universality problem, polarization puzzle, hierarchy problem, negative cross section, large perturbative corrections…

➢ Theoretical: rigorousness of NRQCD

Nayak, Qiu, Sterman, 0509021
Bodwin, Chung, Ee, Kim, Lee, 1910.05497
Zhang, Meng, YQM, Chao, 2011.04905

- Based on EFT of QCD; Factorization has been tested to NNLO

Rigorous QCD analysis of inclusive annihilation and production of heavy quarkonium

Geoffrey T. Bodwin (Argonne), Eric Braaten (Fermilab), G.Peter Lepage (Cornell U., LNS)
Jul, 1994

126 pages
Published in: *Phys.Rev.D* 51 (1995) 1125-1171, *Phys.Rev.D* 55 (1997) 5853 (erratum)
e-Print: hep-ph/9407339 [hep-ph]
DOI: 10.1103/PhysRevD.55.5853, 10.1103/PhysRevD.51.1125
Report number: ANL-HEP-PR-94-24, FERMILAB-PUB-94-073-T, NUHEP-TH-94-5
View in: ADS Abstract Service, OSTI Information Bridge Server, KEK scanned document

pdf links cite claim reference search 2,732 citations

➢ What is missing?

- Remember: summation over all possible n in NRQCD formula
- One possibility: corrections at high powers in v (relativistic corrections)!

Outline

I. Introduction of quarkonium

II. CSM and CEM

III. NRQCD factorization theory

IV. Soft gluon factorization

V. Recent progresses and difficulties

VI. Summary

Relativistic corrections in NRQCD

➢ Relativistic (power) corrections

- **Equations of motion of NRQCD EFT:** $\left(iD_0 - \frac{\boldsymbol{D}^2}{2m} + \cdots\right)\psi = 0$
- **NRQCD factorization: use EOM to remove ∇_0, leaving operators like:**

(Warning: here D replaced by ∇, needs proper gluon fields to make them gauge invariant)

$\chi^\dagger\psi, \chi^\dagger\sigma^i\psi, \chi^\dagger T^a\psi, \chi^\dagger\sigma^i T^a\psi$

Type 0: Different colors, spins

$\chi^\dagger gE^i\psi$

Type 1: Intrinsic gluons (E, B fields)

$\chi^\dagger \overleftrightarrow{\nabla}^i\psi$

Type 2: Orbital angular momentum

$\chi^\dagger \overleftrightarrow{\nabla}^2\psi$

Type 3: Relative momentum

$\nabla^i\left(\chi^\dagger\psi\right)$

Type 4: Total momentum

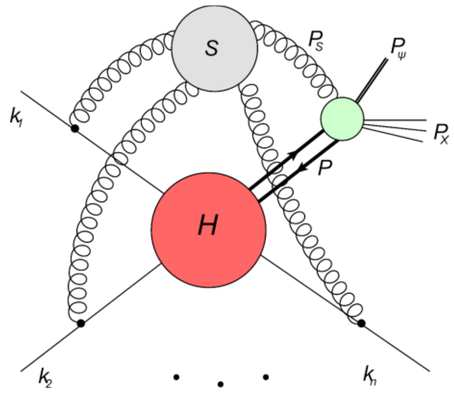
Corrections in type 3 widely studied, for charmonium production in pp collision, about 30%-50% corrections

CS-channel:
Fan, YQM, Chao, 0904.4025
CO-channel:
Xu, Li, Liu, Zhang,1203.0207
S-D mixing-channel:
He, Kniehl, 1507.03882
LP in p_T, all order in v:
Li, Chen, Huang, YQM, 1909.03554

However, more relativistic-correction terms may be needed!

Soft gluon emission

➢ Soft gluon emission in color-bleaching process



YQM, Vogt, 1609.06042

- P_ψ is different from P, $P = P_\psi[1 + O(\lambda)]$
- NRQCD expand P around P_ψ

➢ Bad convergence of NRQCD expansion

- Cross section approximately $\propto P^{-4} = P_\psi^{-4}[1 + O(\lambda)]^{-4}$

Mangano, Petrelli, 9610364

$$\int_{-1}^{1} \frac{d\cos\theta}{2(1+\lambda+\lambda\cos\theta)^4} = 0.42 = 1 - 4\lambda + \frac{40}{3\lambda^2} - 40\lambda^3 + \cdots$$

$$= 1 - 1.2 + 1.2 - 1.08 + 0.91 - 0.73 + \cdots$$

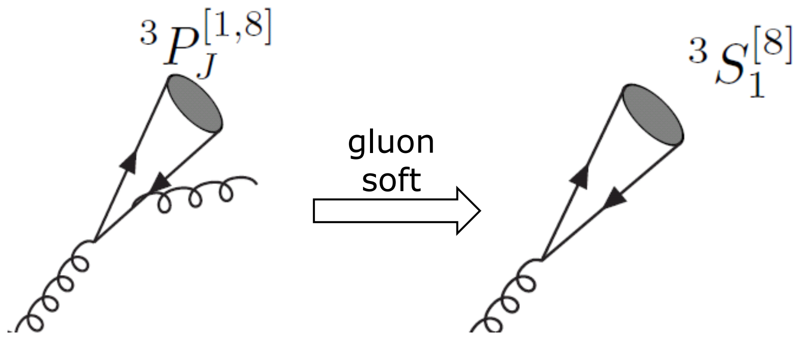
With $\lambda \approx v^2 \approx 0.3$

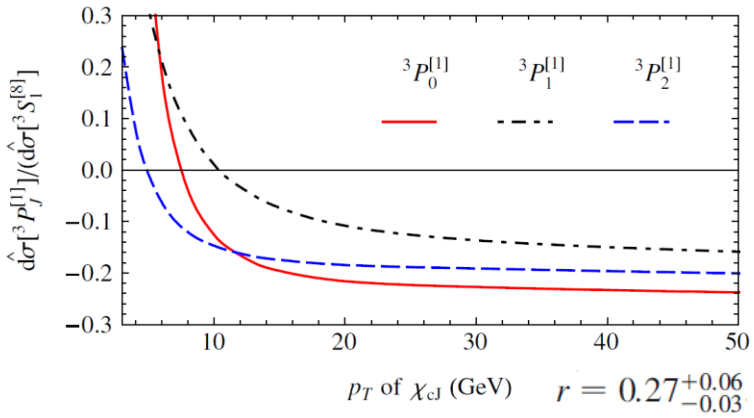
- Solution: soft gluon momentum should be kept but not expanded, which means to resum relativistic corrections (due to kinematic effects) to all powers in v!

Over subtraction

➢ **Eg. χ_{cJ} production:** $d\sigma_{\chi_{cJ}}/(2J+1) \approx d\hat{\sigma}_{^3P_J^{[1]}} \langle O\left(^3P_0^{[1]}\right)\rangle + d\hat{\sigma}_{^3S_1^{[8]}} \langle O\left(^3S_1^{[8]}\right)\rangle$

Braaten, Chen, 9610401
YQM, Wang, Chao, 1002.3987





- **Soft gluon in P-wave: factorized to S-wave matrix element**
- **Subtraction scheme: at *zero momentum*, which contributes the largest production rate. Over subtracted! P-wave negative!**
- **Big cancellation between S-wave and P-wave! Perturbation unstable**
- **Solution: soft gluon momentum should be kept during subtraction process, or resum kinematic effects to all powers in v.**

Threshold region

➢ At threshold region

- Large logarithms appear: can be resummed by introducing shape functions

Beneke, Rothstein, Wise, 9705286
Fleming, Leibovich, Mehen, 0306139
Leibovich, Liu, 0705.3230

- Soft gluon momentum: has leading contribution for quarkonium momentum distribution, cannot be ignored

➢ Combination of logs and powers resummation needed

- Keep soft gluon momentum unexpanded is the first step.

Comments

➢ Relativistic corrections with fixed power in v

- Bad convergence, too many terms are needed
- Involves too many LDMEs, very hard to fix them
- Solution: resum all LDMEs to obtain a function!

  (Like resum twist-2 local operators to obtain PDFs)

➢ What do we need to resum?

- Type 0 ($\chi^\dagger\psi, \chi^\dagger\sigma^i\psi, \chi^\dagger T^a\psi, \chi^\dagger\sigma^i T^a\psi$): finite number, can be studied exclusively
- Type 1-2 insertion ($\chi^\dagger g E^i\psi$, $\chi^\dagger \overleftrightarrow{\nabla}^i \psi$): usually not enhanced, less important
- Type 3 and 4 insertion ($\chi^\dagger \overleftrightarrow{\nabla}^2\psi$, $\nabla^i\left(\chi^\dagger\psi\right)$) : kinematic effects, enhanced if the observable has a steep distribution. E.g., $p_T$distribution in pp collision, momentum distribution in endpoint region.

Soft gluon factorization

Ma, Chao, 1703.08402
Chen, Ma, 2005.08786

➢ Different way to use EoM in NRQCD EFT

- NRQCD factorization: use EOM to remove ∇_0
- SGF: remove relative derivatives $\overleftrightarrow{\nabla}_0, \overleftrightarrow{\nabla}^2$, leaving only total derivatives

➢ Factorization formula

$P = P_H + P_X$: momentum of $Q\bar{Q}$

$$(2\pi)^3 2P_H^0 \frac{d\sigma_H}{d^3P_H} \approx \sum_n \int \frac{d^4P}{(2\pi)^4} \mathcal{H}_n(P) F_{n\to H}(P, P_H)$$

- $\mathcal{H}_n$: perturbatively calculable hard parts
- $F_{n\to H}$: nonperturbative soft gluon distributions (SGDs)
- Comparing with NRQCD factorization: a series of relativistic corrections resummed
- Relation: CO->TMD, NRQCD->SGF

Outline

Preliminary: universality in SGF

➢ **Application to $e^+e^- \to J/\psi({}^3P_J^{[8]}, {}^1S_0^{[8]}) + X$**

- **Partonic differential cross sections**

Chen, Jin, Ma, Meng, 2201.04492

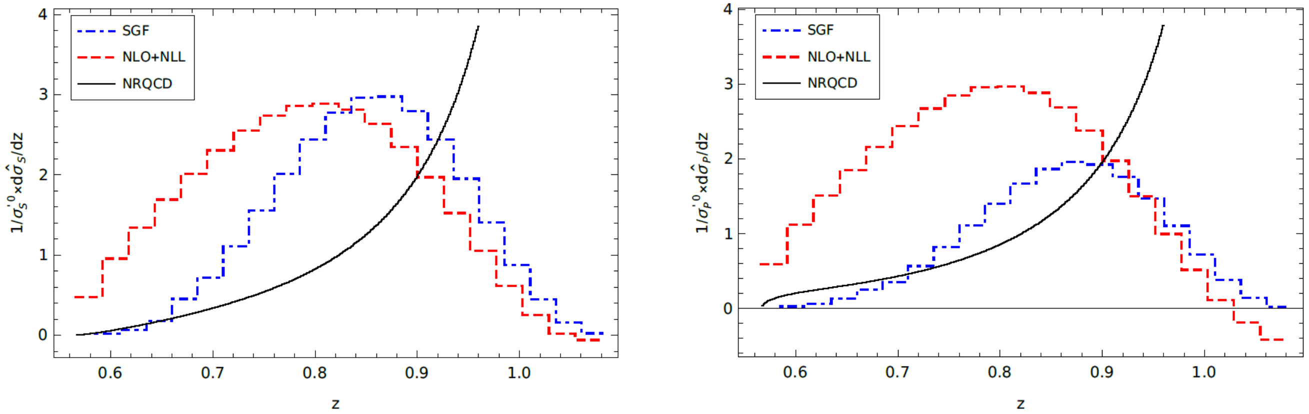


Figure 4. The differential cross sections in SGF and NRQCD factorization approaches.

- **Smaller partonic cross section, larger LDMEs allowed**

$$M_k^X = \langle \mathcal{O}^{J/\psi}({}^1S_0^{[8]})\rangle + k\frac{\langle \mathcal{O}^{J/\psi}({}^3P_0^{[8]})\rangle}{m_c^2}$$

$$\begin{aligned} M_{3.9}^{\text{NRQCD}} &< (2.4 \pm 0.7) \times 10^{-2}\,\text{GeV}^3, \\ M_{3.9}^{\text{NLO+NLL}} &< (5.8 \pm 1.8) \times 10^{-2}\,\text{GeV}^3, \\ M_{2.5}^{\text{SGF}} &< (7.2 \pm 2.2) \times 10^{-2}\,\text{GeV}^3. \end{aligned}$$

- **LDMEs in e^+e^- can be consistent with that extracted in pp**

$$pp:\ \mathrm{M}_0 = \langle O({}^1S_0^{[8]})\rangle + 3.9\langle O({}^3\boldsymbol{P}_0^{[8]})\rangle/m_c^2 \approx (7.4 \pm 1.9) \times 10^{-2}\,\textbf{GeV}^3$$

Fragmentation functions in SGF

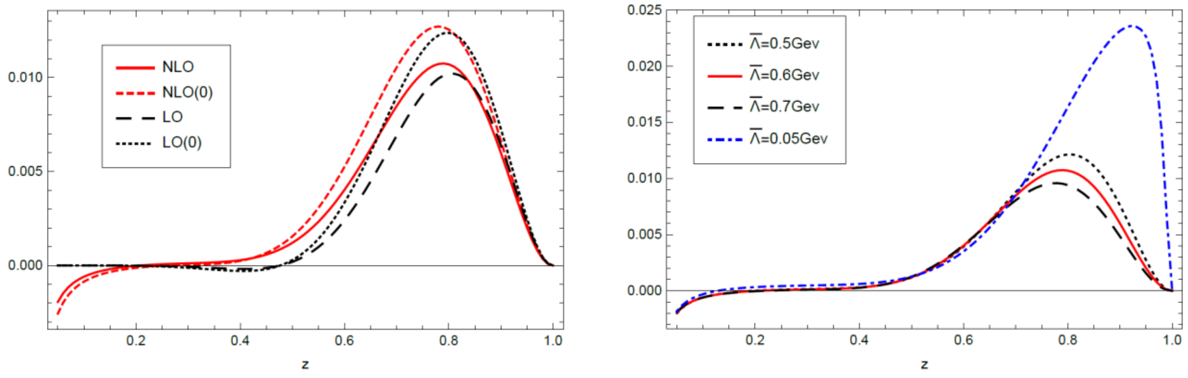
➢ Gluon FFs $g \to Q\overline{Q}\,({}^3S_1^{[8]}) + X$

Chen, Jin, YQM, Meng, 2103.15121

$\bar{\Lambda}$: average momentum emitted

$$D_{g\to H}(z, M_H, m_Q, M_H) = \int_z^1 \frac{dx}{x} \hat{D}_{[SS]}(\hat{z}, M_H/x, m_Q, M_H, M_H) \times F_{[SS]\to H}(x, M_H, m_Q, M_H),$$

$$D^{(0)}_{g\to H}(z, M_H, m_Q, M_H) = \int_z^1 \frac{dx}{x} \hat{D}^{(0)}_{[SS]}(\hat{z}, M_H/x, M_H, M_H) \times F_{[SS]\to H}(x, M_H, m_Q, M_H).$$



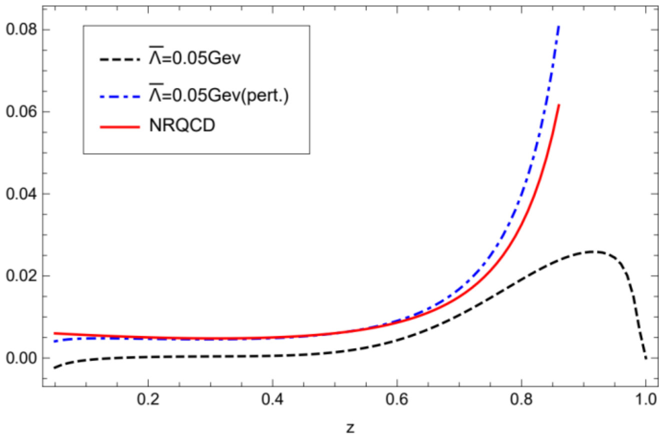


Figure 7. Left figure: Comparison of the gluon FF obtained in different approximations. Rig
figure: $\bar{\Lambda}$ dependence of gluon FF at NLO.

$$R^X(n) \equiv \frac{\int_0^1 dz z^n D^X_{g\to H}(z, M_H, m_Q, \mu)}{\int_0^1 dz z^n D_{g\to H}(z, M_H, m_Q, \mu)},$$

$R^{NRQCD} \approx 6$

Fragmentation functions in SGF

Chen, YQM, Meng, 2304.04552

- **Gluon FFs $g \to Q\overline{Q}\,({}^3\mathrm{P}_J^{[1]}) + X$**
  - In NRQCD: plus-function result in negative results

$$\hat{d}^{(2)}_{g\to {}^3P_J^{[1]}} = \frac{4}{9N_c}\left\{\left[\frac{Q_J}{2J+1} - \frac{1}{2}\,\ln\left(\frac{\mu_\Lambda^2}{4m_Q^2}\right)\right]\delta(1-z) + \frac{z}{(1-z)_+} + \frac{P_J(z)}{2J+1}\right\}$$

  - In SGF: plus-functions are factorized into nonperturbative functions, can be positive

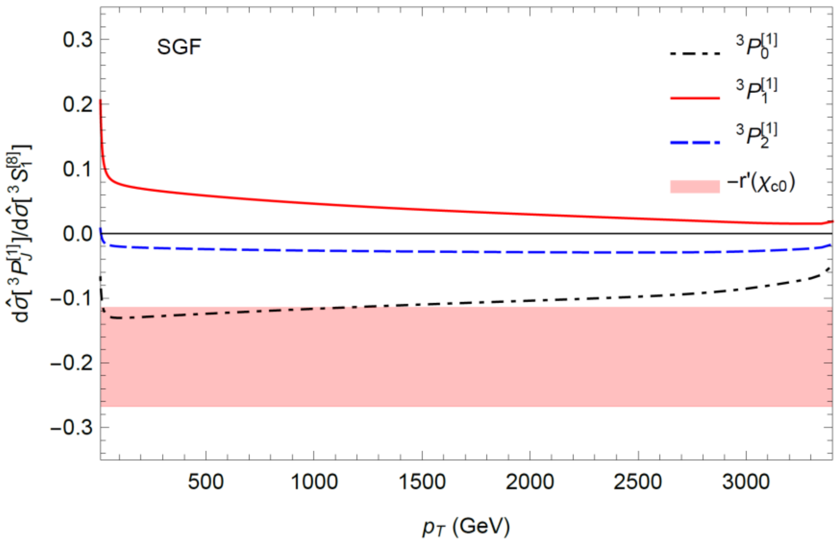
$$\hat{D}^{LO,(0)}_{g\to Q\bar{Q}[{}^3P_0^{[1]}]} = \frac{32\alpha_s^2}{M_H^5 N_c}\frac{2}{9}\left[\frac{1}{36}z(837-162z+72z^2+40z^3+8z^4) + \frac{9}{2}(5-3z)\ln(1-z)\right]$$

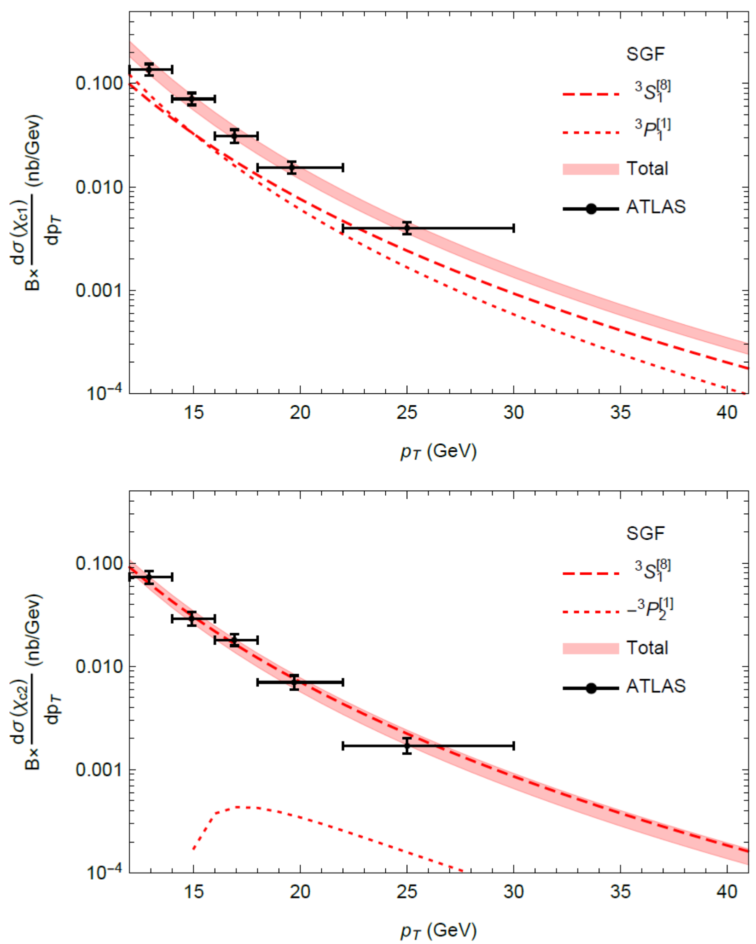
Resolve negative cross section at high p_T

Chen, YQM, Meng, 2304.04552

➢ χ_{cJ} production

- $\chi^2/d.o.f = 0.63/8$, as good as NRQCD
- No substantial cancellations
- Cross sections are positive at high p_T





- See also resummation within NRQCD framework

Chung, 2303.17240

Resolve negative total cross section

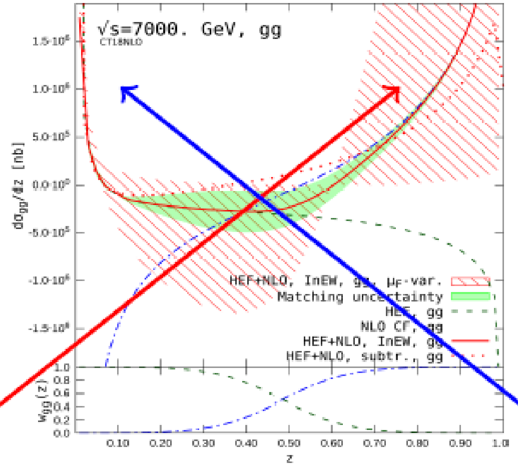
- **Resum** large log $\ln(S/m^2)$ using high energy factorization

Lansberg, Nefedov, Ozcelik, 2112.06789

See also: YQM, Venugopalan, 1408.4075

η_c**-hadroproduction,**

$z = M^2/\hat{s}$:

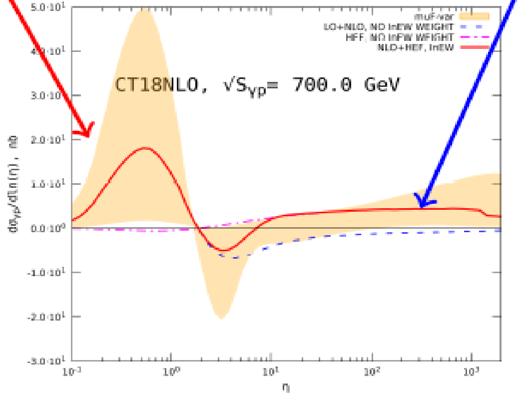


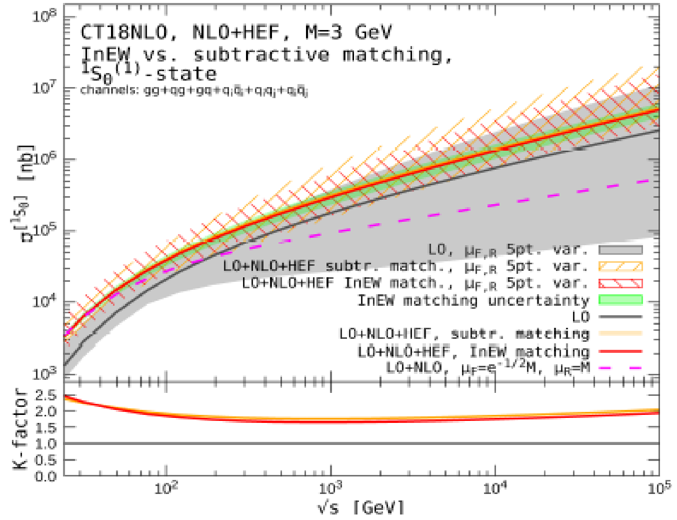
NLO

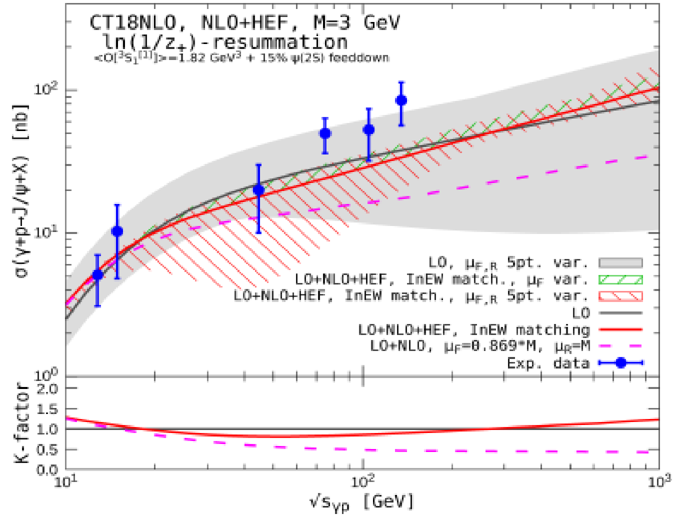
HEF

J/ψ**-photoproduction,**

$\eta = (\hat{s} - M^2)/M^2$:





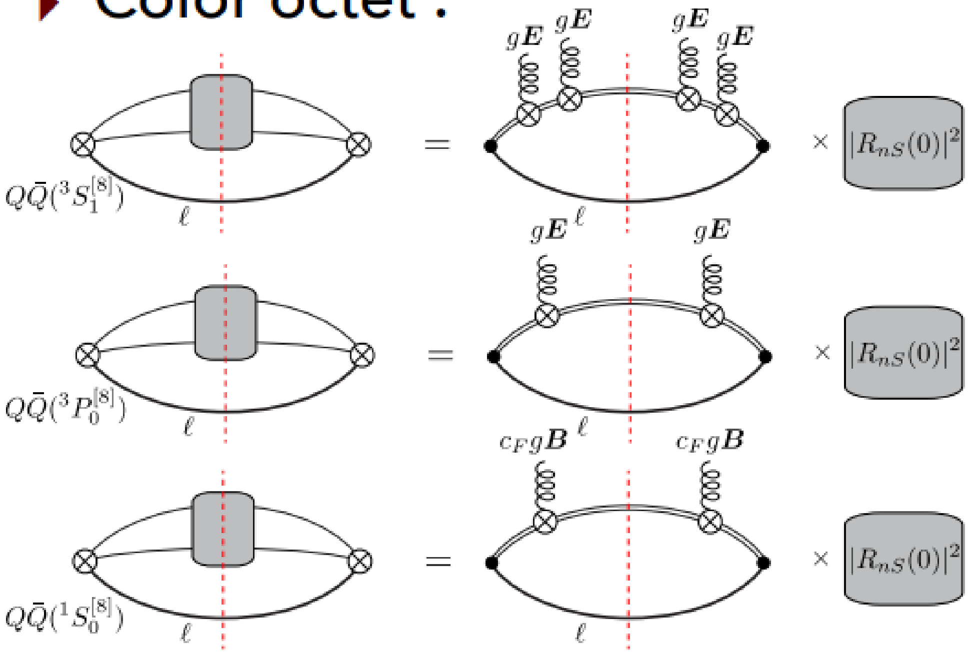


S-WAVE MATRIX ELEMENTS IN PNRQCD

- $V = J/\psi,\ \psi(2S),\ \Upsilon(nS)$. Color singlet : $\langle \mathcal{O}^V(^3S_1^{[1]})\rangle = \frac{3|R(0)|^2}{4\pi}$

$$\sigma_{V+X} = \hat{\sigma}_{Q\bar{Q}(^3S_1^{[1]})}\langle \mathcal{O}^V(^3S_1^{[1]})\rangle + \hat{\sigma}_{Q\bar{Q}(^3S_1^{[8]})}\langle \mathcal{O}^V(^3S_1^{[8]})\rangle + \hat{\sigma}_{Q\bar{Q}(^1S_0^{[8]})}\langle \mathcal{O}^V(^1S_0^{[8]})\rangle + \sum_{J=0,1,2} \hat{\sigma}_{Q\bar{Q}(^3P_J^{[8]})}(2J+1)\langle \mathcal{O}^V(^3P_0^{[8]})\rangle$$

- Color octet :



$$\langle \mathcal{O}^V(^3S_1^{[8]})\rangle = \frac{1}{2N_c m^2}\frac{3|R(0)|^2}{4\pi}\mathcal{E}_{10;10}$$

$$\langle \mathcal{O}^V(^3P_0^{[8]})\rangle = \frac{1}{18N_c}\frac{3|R(0)|^2}{4\pi}\mathcal{E}_{00}$$

$$\langle \mathcal{O}^V(^1S_0^{[8]})\rangle = \frac{1}{6N_c m^2}\frac{3|R(0)|^2}{4\pi}c_F^2\mathcal{B}_{00}$$

Significantly simplified

- ***Three correlators $\mathcal{E}_{10;10}$, $\mathcal{E}_{00}$, $\mathcal{B}_{00}$ to rule all S-wave production***

CROSS SECTION RATIOS

- Universality of the gluonic correlators leads to predictions for cross section ratios, independently of the correlators

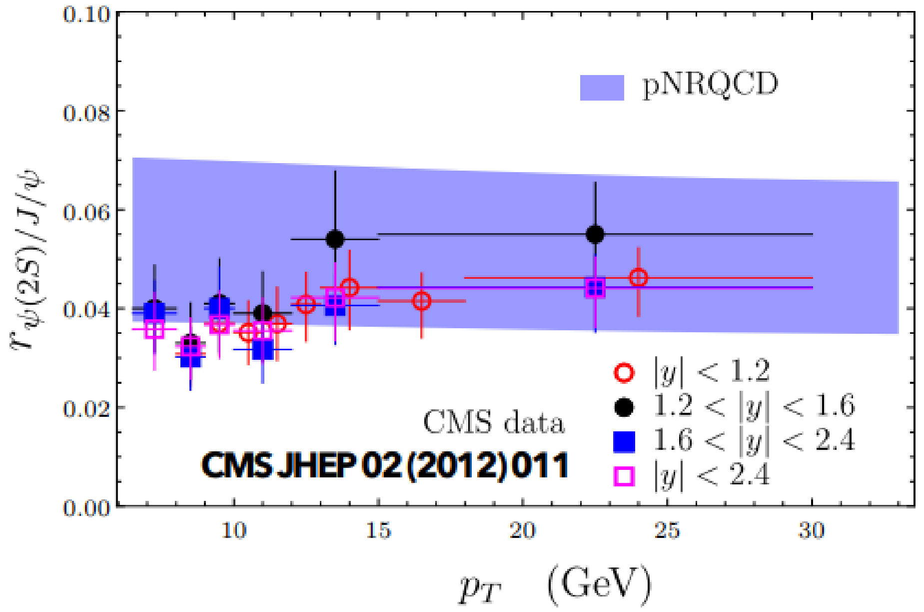
$$\frac{\sigma^{\rm direct}_{\psi(2S)}}{\sigma^{\rm direct}_{J/\psi}} = \frac{|R^{(0)}_{\psi(2S)}(0)|^2}{|R^{(0)}_{J/\psi}(0)|^2} \qquad \frac{\sigma^{\rm direct}_{\Upsilon(3S)}}{\sigma^{\rm direct}_{\Upsilon(2S)}} = \frac{|R^{(0)}_{\Upsilon(3S)}(0)|^2}{|R^{(0)}_{\Upsilon(2S)}(0)|^2}$$

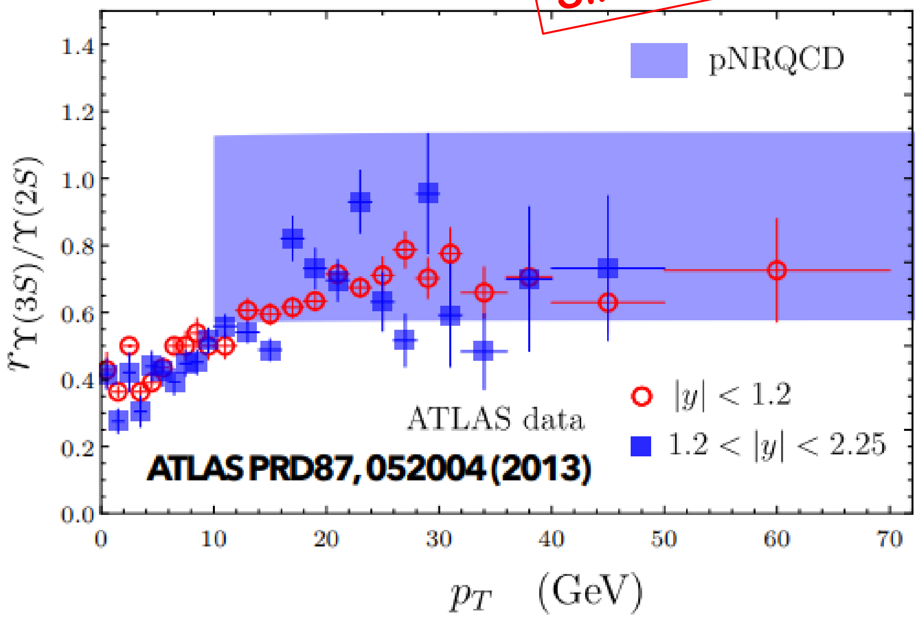
Brambilla, HSC, Vairo, Wang, PRD105, L111503 (2022)

- Compared to experiment, including feeddown effects:

$r_{A/B} = ({\rm Br}_{A\to\mu^+\mu^-}\sigma_A)/({\rm Br}_{B\to\mu^+\mu^-}\sigma_B)$

Similar to CEM, can be improved





DETERMINATIONS OF GLUONIC CORRELATORS

- The fits constrain $\mathcal{E}_{10;10}$ and $\mathcal{E}_{00}$ to be positive, and $\mathcal{B}_{00}$ is small.

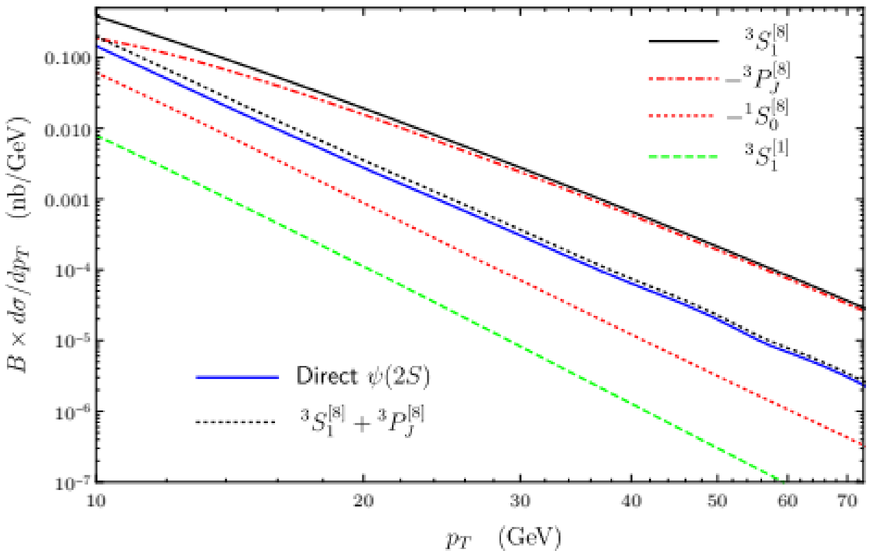
$\langle \mathcal{O}^{J/\psi}({}^3S_1^{[8]})\rangle$ (GeV^3)	$\langle \mathcal{O}^{J/\psi}({}^1S_0^{[8]})\rangle$ (GeV^3)	$\langle \mathcal{O}^{J/\psi}({}^3P_0^{[8]})\rangle/m^2$ (GeV^3)
$(1.40 \pm 0.42) \times 10^{-2}$	$(-0.63 \pm 3.22) \times 10^{-2}$	$(2.59 \pm 0.83) \times 10^{-2}$

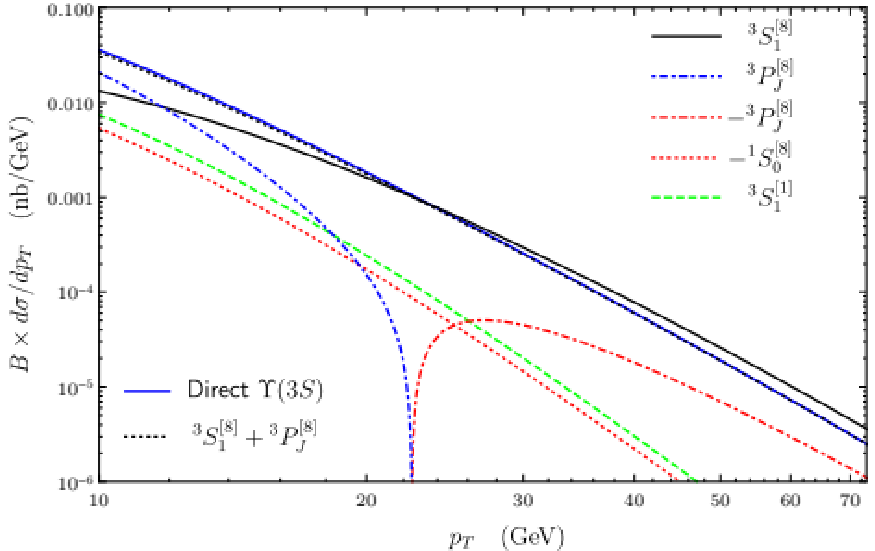
$\frac{p_T}{2m} > 5$

- These also determine $\psi(2S)$ and Υ matrix elements.
- S-wave production is dominated by the ${}^3S_1{}^{[8]}$ + ${}^3P_J{}^{[8]}$.

  Large cancellation occur between ${}^3S_1{}^{[8]}$ and ${}^3P_J{}^{[8]}$ channels.

Agree with pure NRQCD analysis



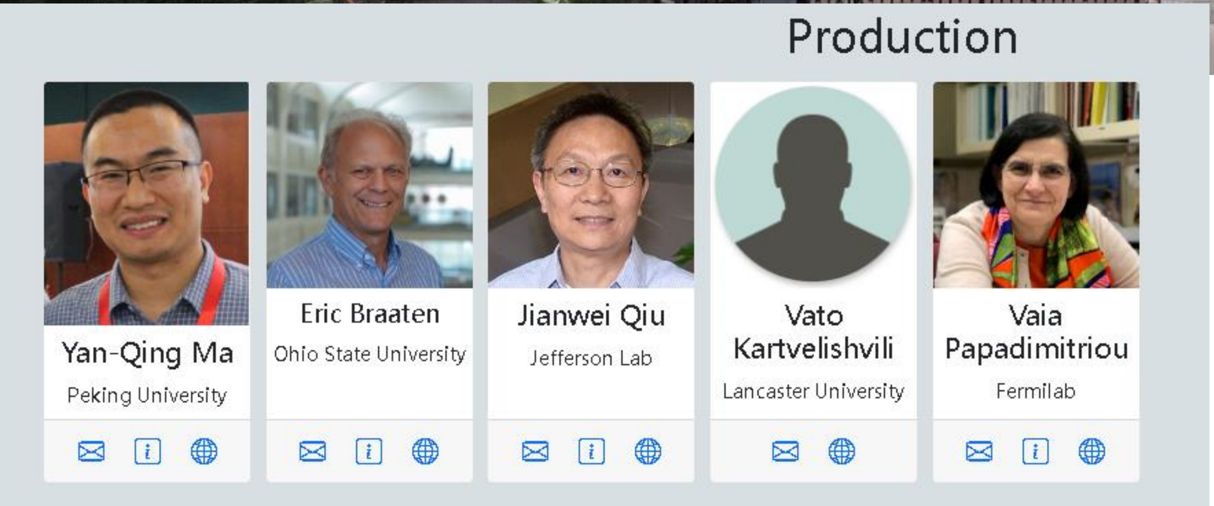


Summary

- Current difficulties: polarization puzzle, hierarchy problem, universality problem, negative cross sections, J/ψ-pair puzzle, high-order puzzle,…
  - Some may be solved in soft gluon factorization
  - Others very hard to understand
- Quarkonium production mechanism: a very important topic
  - But an extremely hard problem
  - New theoretical ideas needed
  - New data needed: confirm previous data; test the energy emitted during hadronization; test the spin symmetry between J/ψ and η_c; …

16TH INTERNATIONAL WORKSHOP ON HEAVY QUARKONIUM (QWG 2024)

IISER MOHALI
FEBRUARY 26, 2024 TO MARCH 1, 2024



Thank you!

Double parton fragmentation function

➢ **Next to leading power (NLP) contribution is important**

- LP contribution of ${}^3S_1^{[8]}$ leads to transversely polarized J/ψ at high p_T, contradicts with data
- At NLO, the LP contributions almost canceled between ${}^3S_1^{[8]}$ and ${}^3P_J^{[8]}$
- LP contribution of ${}^1S_0^{[8]}$ is dominant only for $P_T > 50$ GeV
- LP contribution of ${}^3S_1^{[1]}$ is negligible compared with experimental data

States	Power in v	p_T behavior at LO	p_T behavior at NLO
${}^3S_1^{[1]}$	v^0	p_T^{-8}	p_T^{-6}
${}^3S_1^{[8]}$	v^4	p_T^{-4}(LP)	p_T^{-4}
${}^1S_0^{[8]}$	v^3	p_T^{-6}(NLP)	p_T^{-4}
${}^3P_J^{[8]}$	v^4	p_T^{-6}	p_T^{-4}

Braaten, Doncheski, Fleming, Mangano, 0911.2166
Ma, Wang, Chao, 1009.3655.

To do power expansion first and also to resum large logarithms of p_T^2/m_c^2

Double parton fragmentation function

➢ LP+NLP factorization formalism

Kang, Qiu, Sterman, 1109.1520
Kang, YQM, Qiu, Sterman, 1401.0923

$$
\begin{aligned}
d\sigma_{A+B\to H+X}(p) &\approx \sum_f d\hat{\sigma}_{A+B\to f+X}(p_f = p/z) \otimes D_{H/f}(z, m_Q) \\
&+ \sum_{[Q\bar{Q}(\kappa)]} d\hat{\sigma}_{A+B\to[Q\bar{Q}(\kappa)]+X}(p(1\pm\zeta)/2z, \\
&\quad p(1\pm\zeta')/2z) \otimes \mathcal{D}_{H/[Q\bar{Q}(\kappa)]}(z,\zeta,\zeta',m_Q),
\end{aligned}
$$

$$
p_Q^+ = p^+\frac{1+\zeta}{2z}, \qquad p_{\bar{Q}}^+ = p^+\frac{1-\zeta}{2z},
$$

$$
p_Q'^+ = p^+\frac{1+\zeta'}{2z}, \qquad p_{\bar{Q}}'^+ = p^+\frac{1-\zeta'}{2z}.
$$

- $D_{H/[Q\bar{Q}(\kappa)]}$:double parton FF
- κ represents the pair's color and spin

- Operator definition of double parton FF (example)

$$
\mathcal{D}_{H/[Q\bar{Q}(a8)]}(z,\zeta,\zeta') = \sum_X \int \frac{p^+dy^-}{2\pi} e^{-i(p^+/z)y^-} \int \frac{p^+dy_1^- \, p^+dy_2^-}{(2\pi)^2} e^{i(p^+/2z)(1-\zeta)y_1^-} e^{-i(p^+/2z)(1-\zeta')y_2^-}
$$

$$
\times \frac{4}{(N^2-1)} \langle 0|\psi_i(0)\frac{\gamma^+\gamma_5}{4p^+}(t^a)_{ij}\bar{\psi}_j(y_2^-)|H(p^+)X\rangle\langle H(p^+)X|\psi_l(y^-+y_1^-)\frac{\gamma^+\gamma_5}{4p^+}(t^a)_{lk}\bar{\psi}_k(y^-)|0\rangle,
$$

- NRQCD factorization of FFs

$d_{[Q\bar{Q}(\kappa)]\to[Q\bar{Q}(c)]}$ are available up to $O(\alpha_s)$

$$
\begin{aligned}
D_{H/f}(z, m_Q, \mu) &= \sum_c d_{f\to[Q\bar{Q}(c)]}(z, m_Q, \mu)\langle O^H_{[Q\bar{Q}(c)]}\rangle, \\
\mathcal{D}_{H/[Q\bar{Q}(\kappa)]}(z,\zeta,\zeta',m_Q,\mu), &= \sum_c d_{[Q\bar{Q}(\kappa)]\to[Q\bar{Q}(c)]}(z,\zeta,\zeta',m_Q,\mu)\langle O^H_{[Q\bar{Q}(c)]}\rangle,
\end{aligned}
$$

Ma, Qiu, Zhang, 1501.04556

Double parton fragmentation function

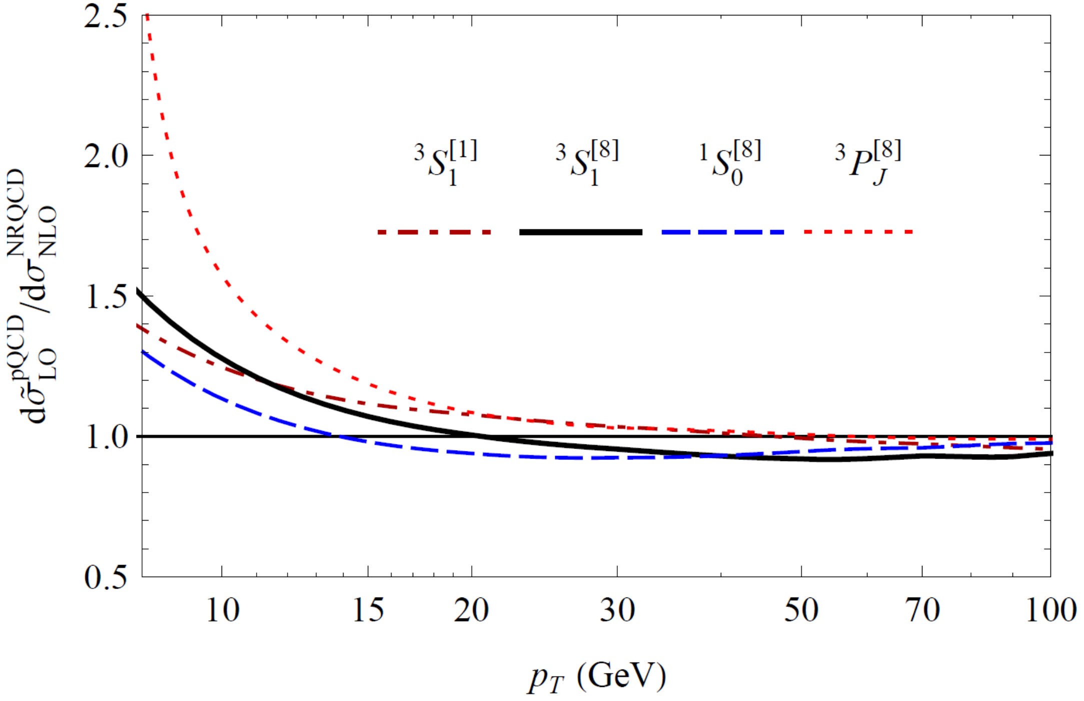
➢ Application to high-P_T quarkonium production

- **Comparison with NLO NRQCD calculations**

Ma, Qiu, Sterman, zhang, 1407.0383

Channel	${}^3S_1^{[1]}$	${}^3S_1^{[1]}$	${}^3S_1^{[8]}$	${}^3S_1^{[8]}$	${}^1S_0^{[8]}$	${}^1S_0^{[8]}$	${}^3P_J^{[8]}$	${}^3P_J^{[8]}$
Power	LP	NLP	LP	NLP	LP	NLP	LP	NLP
PDFs	-	LO	LO	LO	NLO	LO	NLO	LO
FFs	-	α_s^1	α_s^1	α_s^0	α_s^2	α_s^0	α_s^2	α_s^0
SDCs	-	α_s^3	α_s^2	α_s^3	α_s^2	α_s^3	α_s^2	α_s^3

The choices for LP+NLP factorization calculations



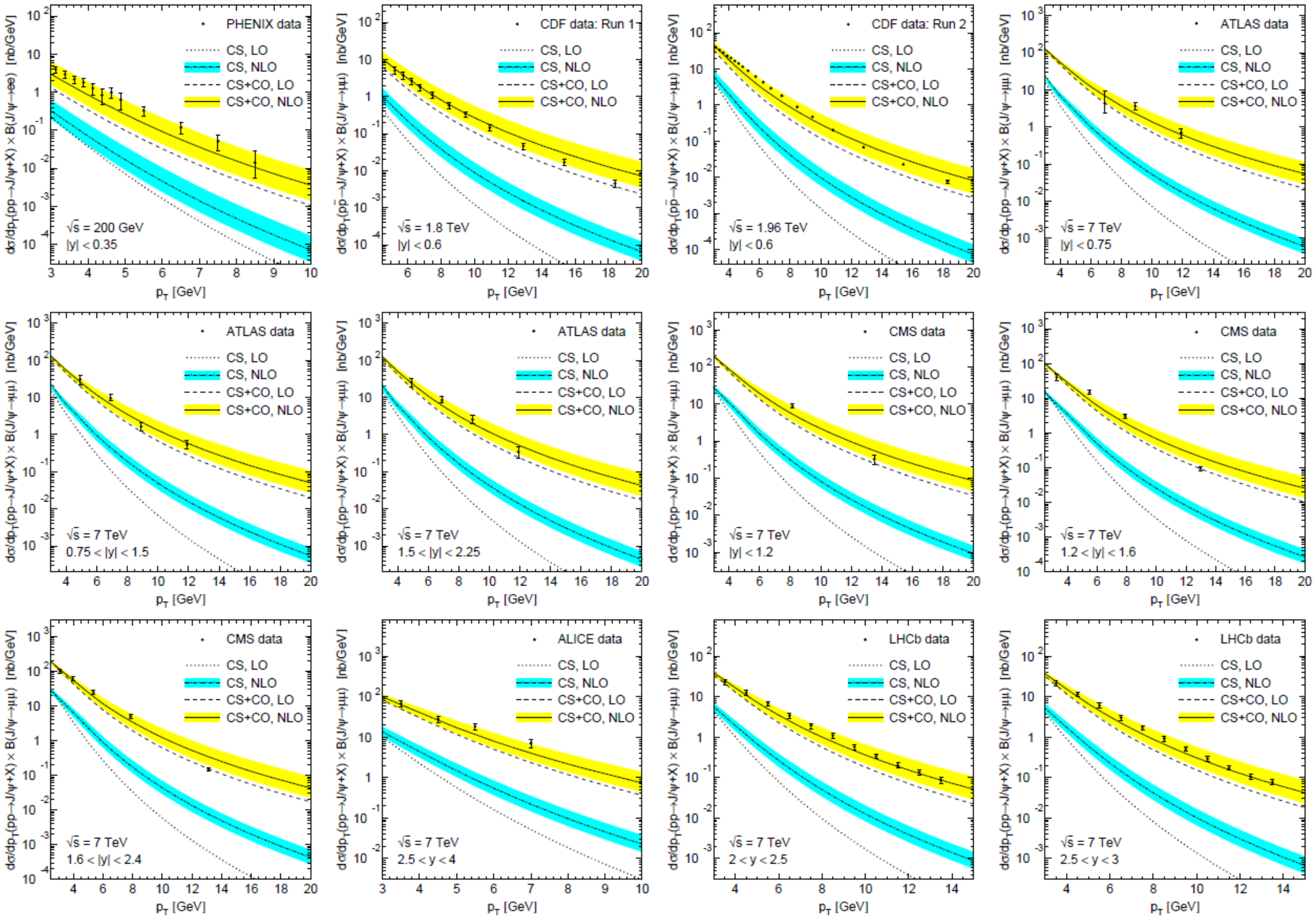
Ratio of J/ψ production rate from LP+NLP factorization over that of NLO NRQCD calculation

LP+NLP factorization calculation naturally reproduces all NLO NRQCD factorization results for high-P_T

Global fit by Butenschoen and Kniehl

➢ **NLO NRQCD V.S. RHIC, Tevatro, LHC data**

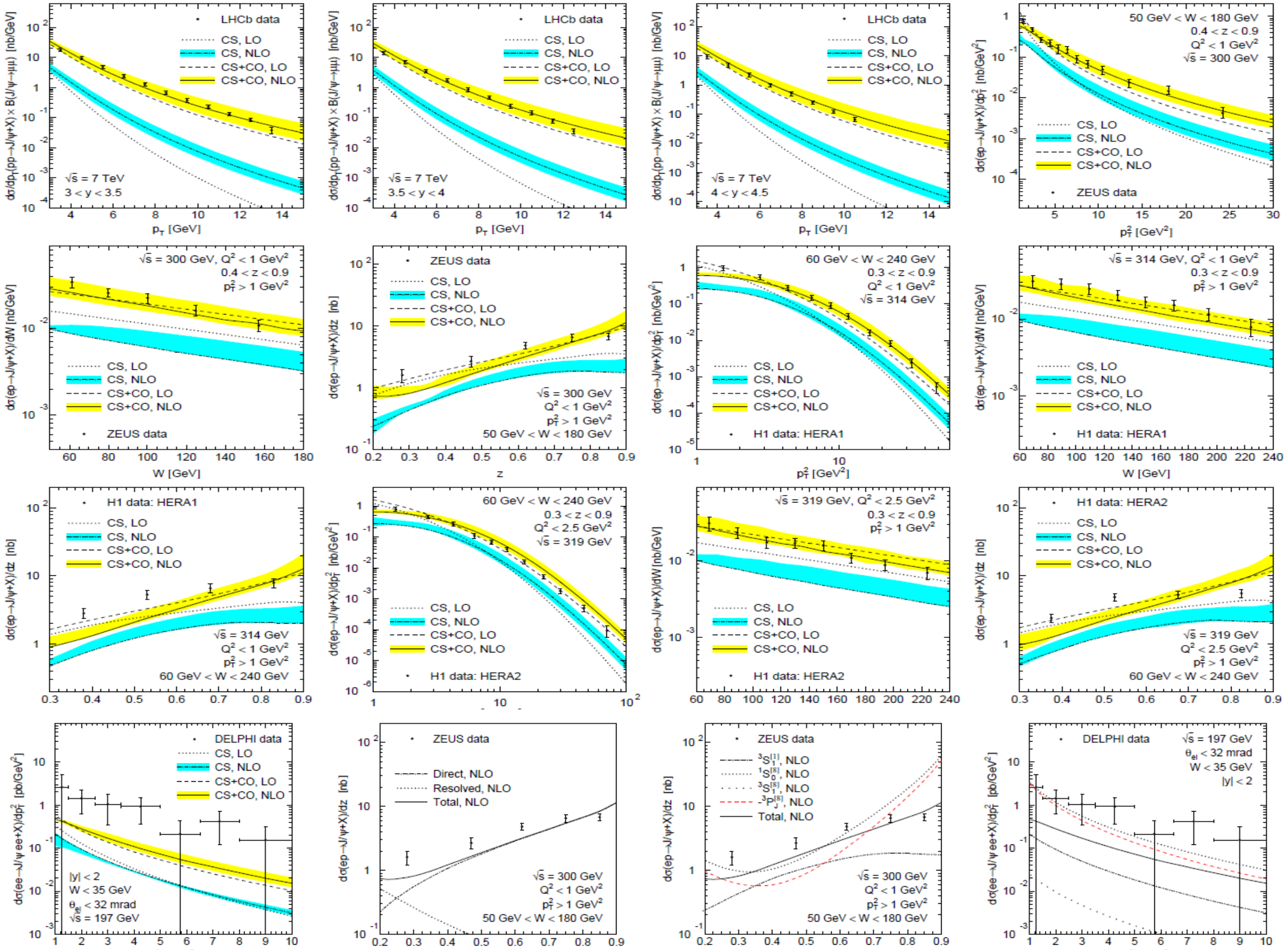
Butenschoen, Kniehl, 1105.0820



Global fit by Butenschoen and Kniehl

➢ **NLO NRQCD V.S. LHC, HERA, LEP data**

Butenschoen, Kniehl, 1105.0820



(a) (b) (c) (d)

NLO fit by ANL-Korea group

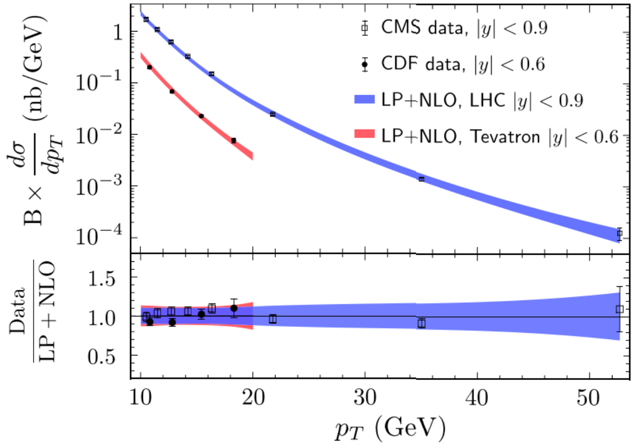
- Fit J/ψ yield data at Tevatron and LHC

Bodwin, Chung, Kim, Lee, 1403.3612

- Exclude $p_T < 10\ GeV$ data
- Large logs at LP in $1/p_T^2$ expansion are resumed

$$\frac{d\sigma^{\rm LP+NLO}}{dp_T} = \frac{d\sigma^{\rm LP}}{dp_T} - \frac{d\sigma^{\rm LP}_{\rm NLO}}{dp_T} + \frac{d\sigma_{\rm NLO}}{dp_T}$$

$$\langle \mathcal{O}^{J/\psi}(^1S_0^{[8]})\rangle = -0.030 \pm 0.381\ \ {\rm GeV}^3$$
$$\langle \mathcal{O}^{J/\psi}(^3S_1^{[8]})\rangle = 0.023 \pm 0.057\ {\rm GeV}^3$$
$$\langle \mathcal{O}^{J/\psi}(^3P_0^{[8]})\rangle = 0.043 \pm 0.106\ {\rm GeV}^5$$



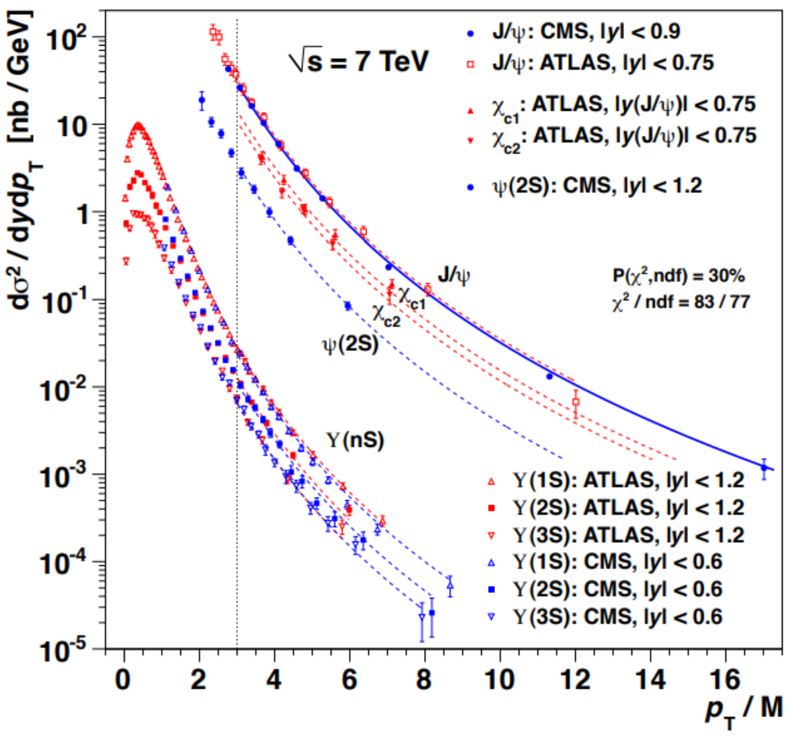
- Good fit, yet another different set of LDMEs

Data driven by CERN et. al. group

➢ Fit J/ψ yield data at Tevatron and LHC

Faccioli, Knunz, Lourenco, Seixas, Wohri, 1403.3970

- Similar shape as functions of p_T/M



- Ignoring ${}^3P_J^{[8]}$ contributions, ${}^1S_0^{[8]}$ dominance

Derivation of SGF: exclusive processes

➢ A different way to use EOM

Li, Feng, YQM, 1911.05886
Chen, YQM, 2005.08786

- NRQCD factorization: use EOM to remove ∇_0
- SGF: remove relative derivatives $\overleftrightarrow{\nabla}_0, \overleftrightarrow{\nabla}^2$, leaving only total derivatives:

$$\langle R^H_{nn_1n_2}\rangle = \langle 0|\nabla_0^{n_1}\nabla^{n_2}(\chi^\dagger \mathrm{K}_n\psi)|H\rangle$$

K_n denotes operators for type 0-2 insertions

- Assume: factorization (similar to NRQCD factorization) is valid to all orders:

$$A^H = \sum_{n,n_1,n_2} \hat{A}_{nn_1n_2}(P_H)\langle R^H_{nn_1n_2}\rangle$$

➢ Using integration by parts

- Remove operators unless $n_1 = n_2 = 0$
- H rest frame: matching coefficients are functions of quarkonium mass

➢ Factorization

$$\mathcal{A}^Q = \sum_n \hat{\mathcal{A}}^n \overline{R}^{n*}_Q \qquad \overline{R}^{n*}_Q = \langle 0|[\overline{\Psi}\mathcal{K}_n\Psi](0)|Q\rangle_S$$

"S": field operators are in small momentum regions

Derivation of SGF: inclusive processes

➢ Use EOM to remove relative derivatives

YQM, Chao, 1703.08402
Chen, YQM, 2005.08786

Resulting: $\langle O^{H+X}_{nn_1n_2n_3n_4}\rangle = \langle 0 \left| \nabla_0^{n_1}\nabla^{n_2}(\chi^\dagger \widetilde{\mathrm{K}}_n\psi)^\dagger \ (a^\dagger_{H+X}a_{H+X})\nabla_0^{n_3}\nabla^{n_4}(\chi^\dagger \mathrm{K}_n\psi) \right| 0\rangle$

where $a^\dagger_{H+X}a_{H+X} \equiv \sum_{J_z^H} |H+X\rangle\langle H+X|$

- Assume: factorization is valid at this level

$$(2\pi)^3 2P_H^0 \frac{d\sigma_H}{d^3P_H} = \sum_{n,n_1,n_2,n_3,n_4} d\hat{\sigma}_{nn_1n_2n_3n_4}(P_H) \sum_{X,P_X} \langle O^{H+X}_{nn_1n_2n_3n_4}\rangle$$

➢ Use integration by parts

- Remove operators unless $n_1 = n_2 = n_3 = n_4 = 0$
- Matching coefficients are functions of (H rest frame): $m_H, E_X, \vec{P}_X^2$

  (Warning: $\vec{P}_X \cdot \langle O^{H+X}_{n0000}\rangle$ is also possible if polarization is concerned)
- In a general frame: $P_H^2, P_H\cdot P_X, P_X^2$

Derivation of SGF: inclusive processes

- **Factorization becomes**

YQM, Chao, 1703.08402
Chen, YQM, 2005.08786

$$(2\pi)^3 2P_H^0 \frac{d\sigma_H}{d^3P_H} = \sum_{n,X,P_X} f(P_H^2, P_H\cdot P_X, P_X^2)\langle O_{n0000}^{H+X}\rangle$$

- **Soft gluon factorization**

$P = P_H + P_X$: **momentum of** $Q\bar{Q}$

$$(2\pi)^3 2P_H^0 \frac{d\sigma_H}{d^3P_H} \approx \sum_n \int \frac{d^4P}{(2\pi)^4} \mathcal{H}_n(P) F_{n\to H}(P, P_H)$$

Production of a heavy quark pair
Expansion in: α_s, (v)

Hadronization (SGDs)
Expansion in: v

- $\mathcal{H}_n$: **perturbatively calculable hard parts**
- $F_{n\to H}$: **nonperturbative soft gluon distributions (SGDs)**
- **UV renormalization scale is suppressed**

$$F_{n\to H}(P,P_H) = \int d^4b e^{-iP\cdot b} \langle 0|[\overline{\Psi}\mathcal{K}_n\Psi]^\dagger(0)\left(a_H^\dagger a_H\right)[\overline{\Psi}\mathcal{K}_n\Psi](b)|0\rangle_S$$

Soft gluon distributions (SGDs)

➢ Operator definition

- Expectation values of bilocal operators in QCD vacuum

$$F_{n\to H}(P,P_H)=\int d^4b e^{-iP\cdot b}\langle 0|[\overline{\Psi}\mathcal{K}_n\Psi]^\dagger(0)\left(a_H^\dagger a_H\right)[\overline{\Psi}\mathcal{K}_n\Psi](b)|0\rangle_S$$

with

$$a_H^\dagger a_H=\sum_X\sum_{J_z^H}|H+X\rangle\langle H+X|$$

$$\mathcal{K}_n(rb)=\frac{\sqrt{M_H}}{M_H+2m}\frac{M_H+\not{P}_H}{2M_H}\Gamma_n\frac{M_H-\not{P}_H}{2M_H}\mathcal{C}^{[c]}$$

Spin project operators: $\Gamma_n=\sum_{L_z,S_z}\langle L,L_z;S,S_z|J,J_z\rangle\Gamma^o_{LL_z}\Gamma^s_{SS_z}$

Color project operators:

$$\mathcal{C}^{[1]}=\frac{\mathbf{1}_c}{\sqrt{N_c}}\qquad\qquad \mathcal{C}^{[8]}=\sqrt{2}t^{\bar{a}}\,\Phi^{(A)}_{a\bar{a}}(rb)$$

Soft gluon distributions (SGDs)

➢ Gauge link

$$\Phi^{(A)}(rb) = \mathcal{P}\exp\left\{-ig_s\int_0^\infty d\lambda\, b_\ell\cdot A^{(A)}(r\,b+\lambda\, b_\ell)\right\}$$

$$b_\ell^\mu = b^\mu + \varepsilon\ell^\mu \qquad\qquad 0 < \varepsilon \ll 1$$

- When b is finite, gauge link along b direction (avoid gauge-link-collinear divergence)
- When $b \to 0$, gauge link unambiguously along l direction (agree with gauge-completed NRQCD matrix elements)

Nayak, Qiu, Sterman, 0509021

➢ Evaluated in *small* region

- Subscript "S": evaluate the matrix element in the region where off-shellness of all particles is much smaller than heavy quark mass

RGEs for SGDs

➢ RGEs

Chen, Jin, YQM, Meng, 2103.15121

$$\frac{d}{d\ln\mu_f}F_{[L'\tilde{L}',\lambda']\to H}(z,M_H,m_Q,\mu_f)=\sum_{L,\tilde{L},\lambda}\int_z^1\frac{dx}{x}\boldsymbol{K}^{[L\tilde{L},\lambda]}_{[L'\tilde{L}',\lambda']}(\hat{z},M_H/x,m_Q,\mu_f)$$
$$\times F_{[L\tilde{L},\lambda]\to H}(x,M_H,m_Q,\mu_f),$$

- **Evolution kernels**

$$\boldsymbol{K}^{[L\tilde{L},\lambda],LO}_{[L'\tilde{L}',\lambda']}(\hat{z},M_H/x,m_Q,\mu_f)=\frac{d}{d\ln\mu_f}F^{NLO}_{[L'\tilde{L}',\lambda']\to Q\bar{Q}[L\tilde{L},\lambda]}(\hat{z},M_H/x,m_Q,\mu_f).$$

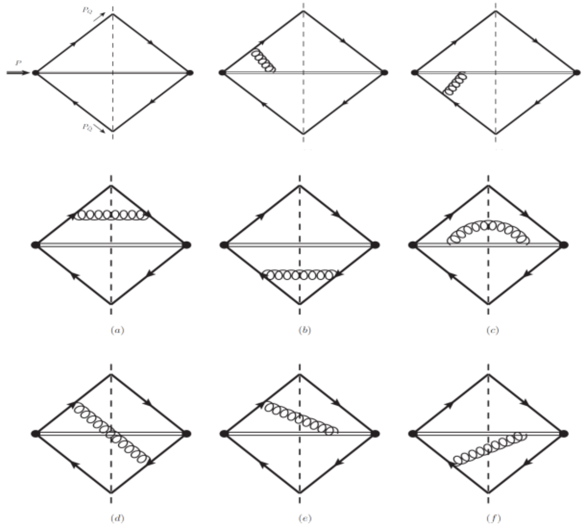
$$\boldsymbol{K}^{[SS],LO}_{[SS]}(z,M_H,m_Q,\mu_f)=\frac{\alpha_s}{\pi}\left\{N_c\left[\frac{2z}{(1-z)_+}-\ln\frac{\mu^2e^{-1}}{M_H^2}\delta(1-z)\right.\right.$$
$$\left.\left.-2\delta(1-z)\left(\frac{1}{2\Delta}\ln\frac{1+\Delta}{1-\Delta}-1\right)\right]+\frac{1}{N_c}\left(\frac{1+\Delta^2}{2\Delta}\ln\frac{1+\Delta}{1-\Delta}-1\right)\delta(1-z)\right\}.$$

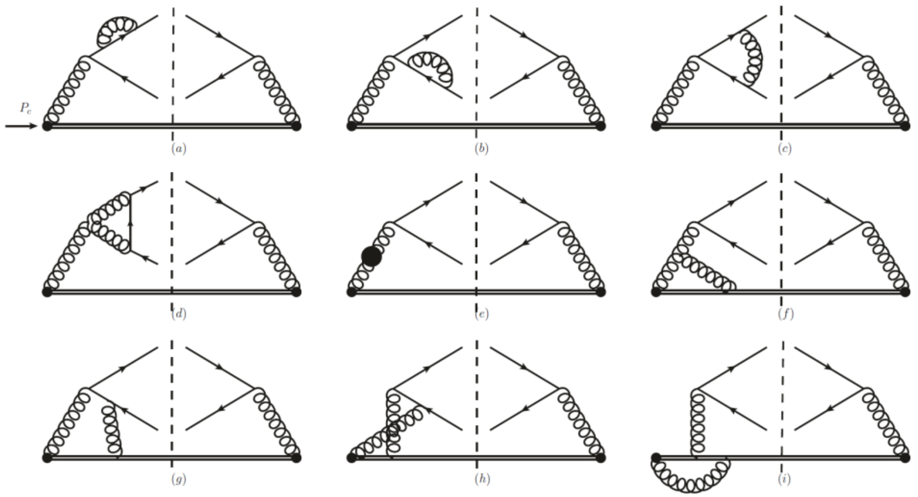
$$\Delta=\frac{\sqrt{M_H^2-4m_Q^2}}{M_H}$$

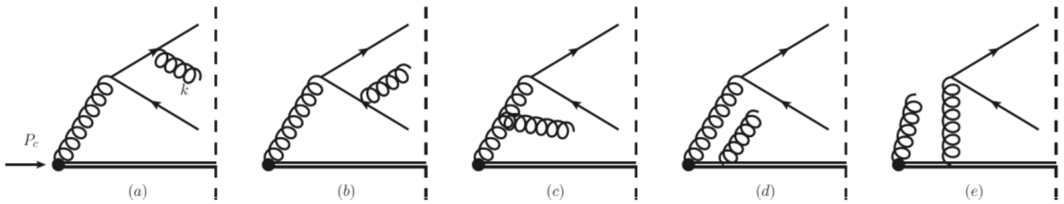
FF: $g \to Q\overline{Q}\,({}^3S_1^{[8]}) + X$ up to NLO

- **Feynman diagrams**

Chen, Jin, YQM, Meng, 2103.15121







Hard part for $g \to Q\overline{Q}\,({}^3S_1^{[8]}) + X$

➢ NRQCD

$$\hat{d}^{(2)}_{g\to {}^3S_1^{[8]}} = \frac{1}{12C_F}\Big[A(\mu_0)\delta(1-z) + \frac{1}{N_c}P_{gg}(z)\Big(\ln\big(\frac{\mu_0^2}{4m_Q^2}\big) - 1\Big) + \frac{2(1-z)}{z} - \frac{4(1-z+z^2)^2}{z}\Big(\frac{\ln(1-z)}{1-z}\Big)_+\Big],$$

Braaten, Lee, 0004228
YQM, Qiu, Zhang, 1311.7078

- **Double logs as $z \to 1$ (threshold logs)**

➢ SGF

Chen, Jin, YQM, Meng, 2103.15121

$$\hat{D}^{LO,(0)}_{[SS]}(\hat{z}, M_H/x, \mu, \mu_f) = \frac{\pi\alpha_s}{(N_c^2-1)}\frac{8x^3}{M_H^3}\delta(1-\hat{z}), \qquad (5.28a)$$

$$\begin{aligned}
&\hat{D}^{NLO,(0)}_{[SS]}(\hat{z}, M_H/x, \mu, \mu_f) \\
&= \frac{4\alpha_s^2 N_c x^3}{(N_c^2-1)M_H^3}\Big[\frac{1}{2}\delta(1-\hat{z})\Big(2A(\mu, M_H/x) + \frac{2\beta_0}{N_c}\ln\Big(\frac{x^2\mu_f^2 e^{-1}}{M_H^2}\Big) + \ln^2\Big(\frac{x^2\mu_f^2 e^{-1}}{M_H^2}\Big) \\
&\quad + \frac{\pi^2}{6} - 1\Big) + \frac{1}{N_c}P_{gg}^{(0)}(\hat{z})\ln\Big(\frac{\mu^2}{\mu_f^2}\Big) + \Big(\frac{2(1-\hat{z})}{\hat{z}} + \hat{z}(4+2\hat{z}^2) + \frac{2\hat{z}^4}{9}(5+\hat{z})\Big) \\
&\quad \times \Big(\ln\Big(\frac{x^2\mu_f^2 e^{-1}}{M_H^2}\Big) - 2\ln(1-\hat{z})\Big) + \frac{2(1-\hat{z})}{\hat{z}} - \Big(\frac{4\hat{z}^4}{1-\hat{z}} - \frac{4\hat{z}^4}{9}(5+\hat{z})\Big)\ln\hat{z}\Big].
\end{aligned} \qquad (5.28b)$$

- **No threshold logs in hard part**
- **Logs are factorized to SGDs and then resummed by using REGs**

Nonperturbative models

➢ The first class of models

$$F^{\rm mod}(\omega') = M_H N_H \frac{b^b}{\Gamma(b)} \frac{\omega'^{b-1}}{\bar\Lambda^b} e^{-b\omega'/\bar\Lambda}, \quad \omega' = M_H(1/x-1),$$ Fleming, Leibovich, Mehen, 0306139

Model-1: $F^{\rm mod}(\omega')|_{\bar\Lambda=0.6{\rm GeV},b=2}$, Model-2: $F^{\rm mod}(\omega')|_{\bar\Lambda=0.6{\rm GeV},b=1}$,

Model-3: $F^{\rm mod}(\omega')|_{\bar\Lambda=0.6{\rm GeV},b=3}$, Model-4: $F^{\rm mod}(\omega')|_{\bar\Lambda=0.5{\rm GeV},b=2}$,

Model-5: $F^{\rm mod}(\omega')|_{\bar\Lambda=0.7{\rm GeV},b=2}$,

the zeroth, first and second moments are $M_H N_H$, $M_H N_H \bar\Lambda$ and $M_H N_H \bar\Lambda^2(\frac{1}{b}+1)$

➢ The other models

Model-6: $4M_H N_H[\theta(w' \geq \frac{19}{40}) - \theta(w' > \frac{29}{40})]$,

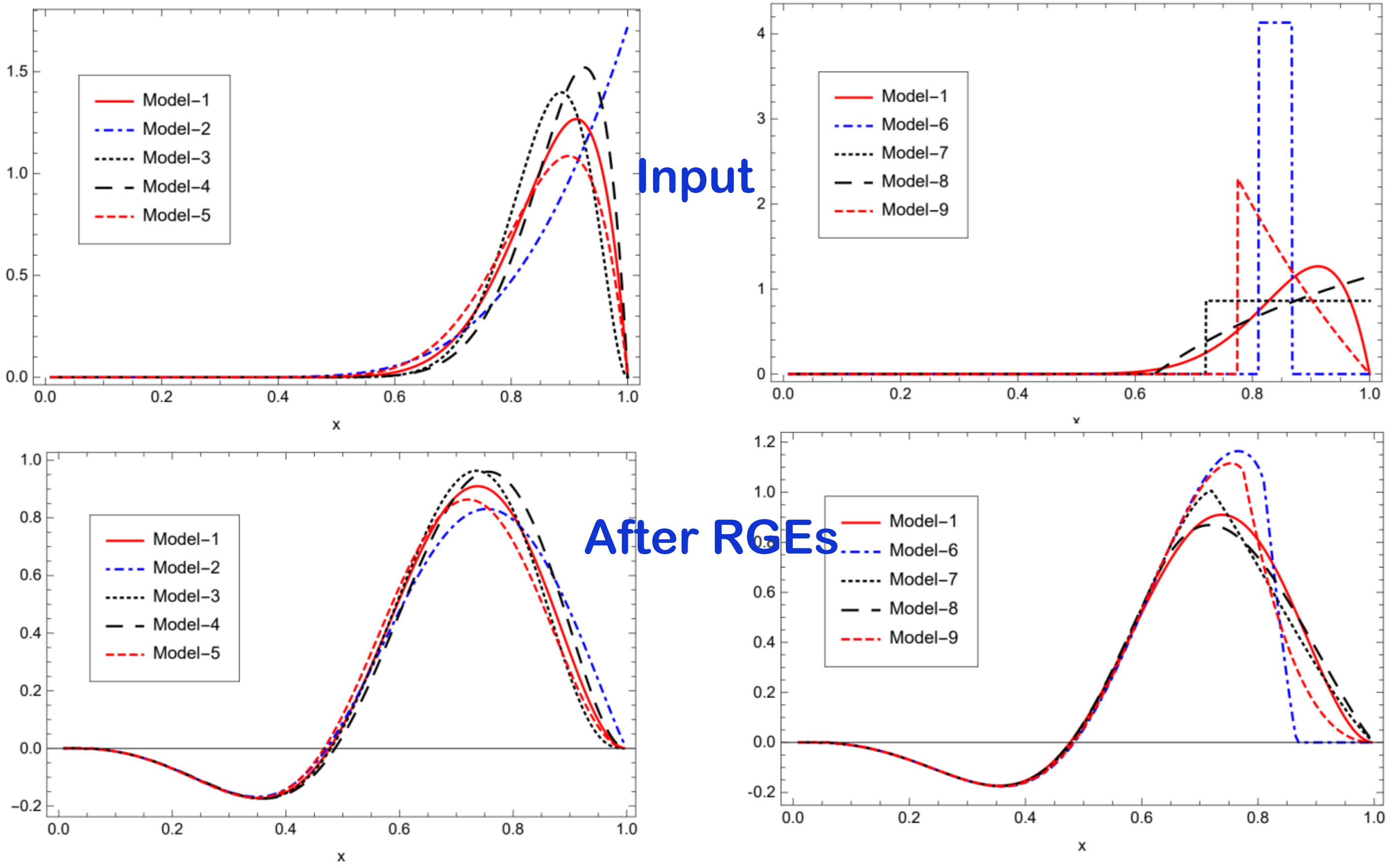
Model-7: $\frac{5}{6} M_H N_H[\theta(w' \geq 0) - \theta(w' > \frac{6}{5})]$,

Model-8: $\begin{cases} M_H N_H(-\frac{50}{81}w' + \frac{10}{9}), & 0 \leq w' \leq \frac{9}{5}, \\ 0, & w' > \frac{9}{5}, \end{cases}$

Model-9: $\begin{cases} \frac{200}{81} M_H N_H w', & 0 \leq w' \leq \frac{9}{10}, \\ 0, & w' > \frac{9}{10}. \end{cases}$

RGEs effects

➢ **Model dependence is significantly reduced after using RGEs**



Numerical: $g \to Q\overline{Q}\,({}^3S_1^{[8]}) + X$

- **Gluon FFs in ${}^3S_1^{[8]}$ channel**

Chen, Jin, YQM, Meng, 2103.15121

$\bar{\Lambda}$: average momentum emitted

$$D_{g\to H}(z, M_H, m_Q, M_H) = \int_z^1 \frac{dx}{x} \hat{D}_{[SS]}(\hat{z}, M_H/x, m_Q, M_H, M_H) \times F_{[SS]\to H}(x, M_H, m_Q, M_H),$$

$$D^{(0)}_{g\to H}(z, M_H, m_Q, M_H) = \int_z^1 \frac{dx}{x} \hat{D}^{(0)}_{[SS]}(\hat{z}, M_H/x, M_H, M_H) \times F_{[SS]\to H}(x, M_H, m_Q, M_H).$$

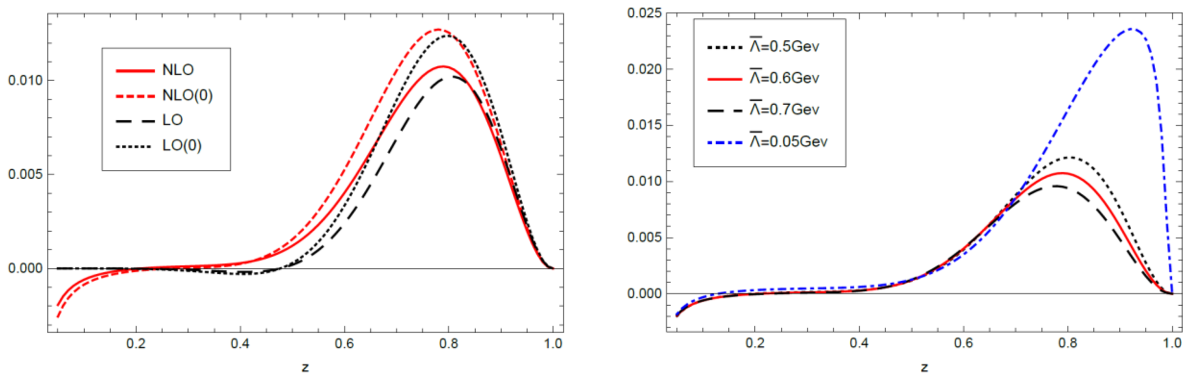
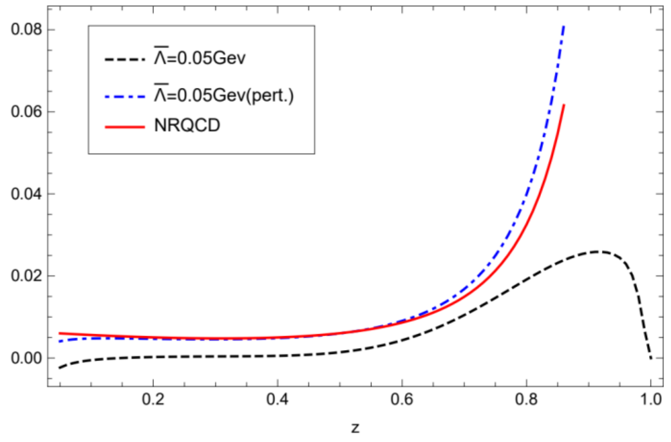


Figure 7. Left figure: Comparison of the gluon FF obtained in different approximations. Rig[ht]
figure: $\bar{\Lambda}$ dependence of gluon FF at NLO.

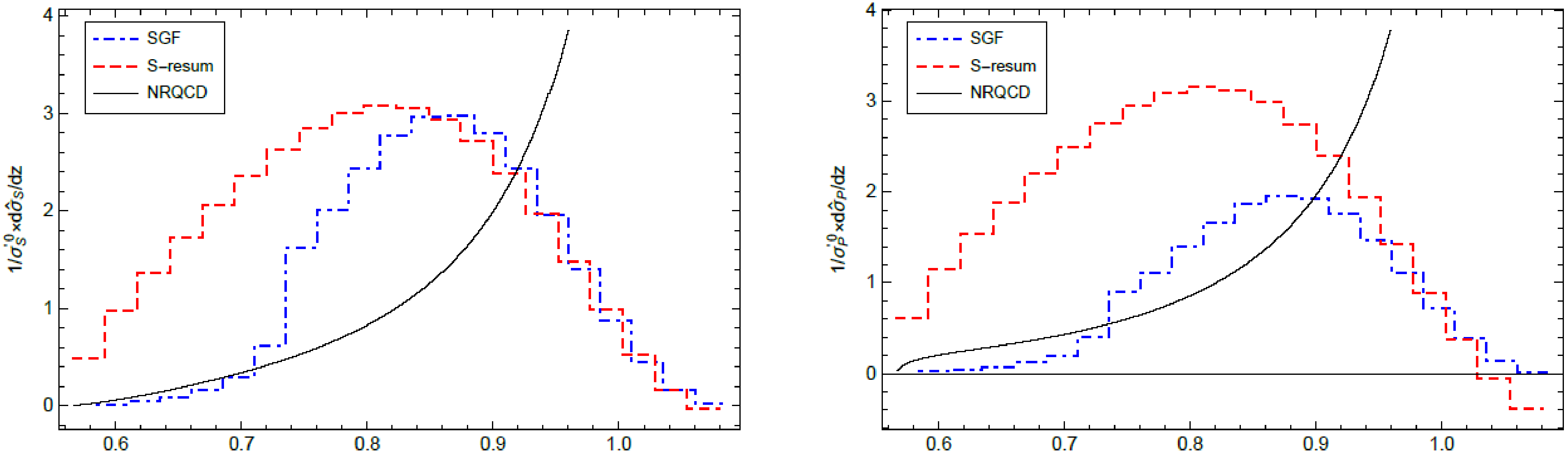


$$R^X(n) \equiv \frac{\int_0^1 dz z^n D^X_{g\to H}(z, M_H, m_Q, \mu)}{\int_0^1 dz z^n D_{g\to H}(z, M_H, m_Q, \mu)},$$

$R^{NRQCD} \approx 6$

J/ψ production at B factories

Chen, Jin, YQM, Meng, in preparation



	NRQCD	NLO+NLL	$\mathcal{S}$-resum	SGF
$\hat{\sigma}_S^{\mathrm{NLO}}$ (pb/GeV^3)	17.677	7.411	8.246	5.712
$\hat{\sigma}_P^{\mathrm{NLO}}$ (pb/GeV^5)	30.370	12.691	14.096	6.375

- Similar to SCET+NRQCD at large z, different at small z
- Has smaller total cross section, easier to reconcile with pp data