Two-color quark matter at nonzero temperature and density

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Hefei, November 3, 2011
1 Motivation

2 QCD-like theories

3 Thermodynamics of two-color QCD

4 Center-symmetric effective theory for two-color QCD

5 Outlook & challenges
Motivation

QCD-like theories

Thermodynamics of two-color QCD

Center-symmetric effective theory for two-color QCD

Outlook & challenges
QCD phase diagram: What we (would like to) know

Low density and high temperature:
- Lattice simulations.
- Heavy ion collisions.

High density and low temperature:
- Lattice simulation not feasible due to sign problem.
- Experimental data inconclusive.

http://www.gsi.de
QCD phase diagram: What we (would like to) know

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High density and low temperature:
- Lattice simulation not feasible due to **sign problem**.
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Are effective models all we can do?
Theorist’s dream

<table>
<thead>
<tr>
<th>Theory</th>
<th>Analytic approach</th>
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Perturbation theory

Freedman, McLerran, PRD 16 (1977) 1130, 1169
Kurkela, Romatschke, Vuorinen, PRD 81 (2010) 105021

QC

No sign problem

Low-energy EFT

Kanazawa, Wettig, Yamamoto, JHEP 08 (2009) 003

Effective models

TB, Fukushima, Hidaka, PRD 80 (2009) 074035
Andersen, TB, PRD 81 (2010) 096004
Zhang, TB, Rischke, JHEP 1006 (2010) 064
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Theory
- Analytic approach
- Lattice simulation

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- **Theory**
  - QCD
    - QC\(_2\)D, aQCD, …

- **Analytic approach**
  - Solve exactly

- **Lattice simulation**
  - Sign problem
    - No sign problem

References:
- Freedman, McLerran, PRD 16 (1977) 1130, 1169
- Kurkela, Romatschke, Vuorinen, PRD 81 (2010) 105021
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No sign problem
Why QC\textsubscript{2}D or aQCD (or similar)?

- No sign problem $\implies$ lattice simulation feasible.
- Use the results to discriminate between the models.
- Baryons in QCD-like theories are bosons!
Why QC$_2$D or aQCD (or similar)?

- No sign problem $\implies$ lattice simulation feasible.
- Use the results to discriminate between the models.
- **Baryons in QCD-like theories are bosons!**

Ideal for model building!

- No annoying three-body physics at low density.
- Gauge-invariant order parameter at high density.
- Decent chance of describing low & high density matter with a single model: a dream of nuclear (astro)physicists!
Why models?

First a bit of pessimism.

- How reliable are the results?
  - Check model dependence: bad news.
  - Check approximation dependence.
- Don’t go too far, it is not worth of the effort.
Why models?

There are some good news too.

- Model calculations are usually economical.
- May be used for a first rough calculation.
- Help to identify interesting problems.
  (Much literature on color superconductivity.)
- Can test ideas used in other approaches.

Andersen, Kyllingstad, Splittorff, JHEP 01 (2010) 055
Outline

1. Motivation
2. QCD-like theories
3. Thermodynamics of two-color QCD
4. Center-symmetric effective theory for two-color QCD
5. Outlook & challenges
**What QCD-like theories do I have in mind?**

<table>
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<th>Pseudoreal theories</th>
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<td>“$\text{QC}_2\text{D}$” = two-color QCD with fundamental quarks.</td>
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<td>(Almost) the same structure is shared by all QCD-like theories with quarks in a pseudoreal representation.</td>
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<tr>
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What makes them interesting? (pseudoreal)

Lattice simulation.
- Determinant of Dirac operator real even at nonzero chemical potential.
- **Even** number of flavors with equal chemical potentials $\implies$ no sign problem.

Spectrum and the phase diagram.
- Baryons are bosons **antisymmetric in color**.
- Nonzero density realized by BEC of diquarks rather than a Fermi sea of nucleons.
- Global symmetry of theory with $N_f$ massless quarks is not $SU(N_f)_L \times SU(N_f)_R \times U(1)_B$, but rather $SU(2N_f)$!
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More about the flavor symmetry (pseudoreal)

The flavor symmetry ...

- $q$ and $\bar{q}$ have the same color transformation properties.
- Exchange $\bigcirc \leftrightarrow \bullet$ does not affect color symmetry.
- Trade $q_R \rightarrow q_L^c \implies$ the theory effectively has $2N_f$ flavors of Weyl fermions, thus $SU(2N_f)$.

...and its consequences.

- $\bigcirc \leftrightarrow \bullet$ is a symmetry of the theory.
- Multiplets of states contain both mesons and diquarks.
- There are diquark NG bosons of $SU(2N_f) \rightarrow Sp(2N_f)$.
- $N_f = 2$: five NG bosons $\pi^0, \pi^\pm, \Delta, \Delta^*$.
- Dense matter in reach of chiral perturbation theory!
More about the flavor symmetry (real)

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- Multiplets of states contain both mesons and diquarks.
- There are diquark NG bosons of \( \text{SU}(2N_f) \rightarrow \text{SO}(2N_f) \).
- \( N_f = 2 \): nine NG bosons \( \pi^0, \pi^\pm, \vec{\Delta}, \vec{\Delta}^* \).
- Dense matter in reach of chiral perturbation theory!
Symmetry-breaking patterns (pseudoreal)

\[ \text{SU}(2N) \rightarrow \text{Sp}(2N) \]

\[ \mu_B \]

\[ \text{SU}(N)_L \times \text{SU}(N)_R \times U(1)_B \rightarrow \text{SU}(N)_V \times U(1)_B \]

\[ m_q \]
Symmetry-breaking patterns (pseudoreal)

\[ \text{SU}(2N) \xrightarrow{m_q} \text{Sp}(2N) \]
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Symmetry-breaking patterns (pseudoreal)

SU(2N) \xrightarrow{m_q} Sp(2N)

SU(N)_L \times SU(N)_R \times U(1)_B \xrightarrow{m_q} SU(N)_V \times U(1)_B

\sigma

\Delta

Sp(N)_L \times Sp(N)_R

Sp(N)_V
Symmetry-breaking patterns (real)

\[ \text{SU}(2N) \rightarrow \text{SO}(2N) \]

\[ \text{SU}(N)_L \times \text{SU}(N)_R \times U(1)_B \rightarrow \text{SU}(N)_V \times U(1)_B \]

\[ \sigma \]

\[ m_q \]

\[ \mu_B \]

\[ \Delta \]
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Model setup

- We use the (P)NJL model in the mean-field approximation:

\[ \mathcal{L} = \bar{\psi}(i\slashed{D} - m_0)\psi + G\left[ (\bar{\psi}\psi)^2 + (\bar{\psi}\gamma_5\tau\psi)^2 + |\bar{\psi}\gamma_5\sigma_2\tau_2\psi|^2 \right] \]

- Diquark and meson couplings same thanks to SU(4).

- Input physical quantities:

<table>
<thead>
<tr>
<th>Physical Quantity</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_c )</td>
<td>270 MeV</td>
</tr>
<tr>
<td>( \sigma_s )</td>
<td>((425 \text{ MeV})^2)</td>
</tr>
<tr>
<td>( \langle \bar{\psi}_u\psi_u \rangle )</td>
<td>((-218 \text{ MeV})^3)</td>
</tr>
<tr>
<td>( f_\pi )</td>
<td>75.4 MeV</td>
</tr>
<tr>
<td>( m_\pi )</td>
<td>140 MeV</td>
</tr>
</tbody>
</table>

- Alternative: O(6) linear sigma model at tree level.

- Improves on LO \( \chi \)PT by including finite \( m_\sigma \) effects.
Phase diagram

TB, Fukushima, Hidaka, PRD 80 (2009) 074035
The insensitivity of deconfinement temperature to $\mu_B$ is an obvious artifact of the PNJL model.
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However, can it still be right?
Chiral regime

- Vacuum: $\langle \bar{\psi}\psi \rangle \neq 0$.
- $\mu_B > m_\pi$: diquark condensation.
- $\langle \bar{\psi}\psi \rangle \neq 0$.

- **LO $\chi$PT at finite $\mu_B$:**
  Kogut et al., NPB 582 (2000) 477

- **Lattice simulation** (staggered adjoint quarks):
  Hands et al., EPJC 17 (2000) 285;
  EPJC 22 (2001) 451

- **NJL model calculation:**
  Ratti, Weise, PRD 70 (2004) 054013
Thermodynamics of BEC transition

\[ \frac{m_\sigma}{m_\pi} = \infty \quad (\chi \text{PT}) \]

- NJL and linear sigma models give identical results.
- Explanation: condensate contribution dominates!
- Do the models stand comparison with lattice?

Andersen, TB, PRD 81 (2010) 096004
Thermodynamics of BEC transition

\[ \frac{m_\sigma}{m_\pi} = 20 \quad (\ell_\sigma m) \]

- NJL and linear sigma models give identical results.
- Explanation: condensate contribution dominates!
- Do the models stand comparison with lattice?

Andersen, TB, PRD 81 (2010) 096004
Thermodynamics of BEC transition

\[ \frac{m_\sigma}{m_\pi} = 15 \quad (\ell_\sigma m) \]

- NJL and linear sigma models give identical results.
- Explanation: condensate contribution dominates!
- Do the models stand comparison with lattice?

Andersen, TB, PRD 81 (2010) 096004
Thermodynamics of BEC transition

\[ \frac{m_\sigma}{m_\pi} = 12 \quad (\ell \sigma m) \]

- NJL and linear sigma models give identical results.
- Explanation: condensate contribution dominates!
- Do the models stand comparison with lattice?

Andersen, TB, PRD 81 (2010) 096004
Thermodynamics of BEC transition

\[ \frac{m_\sigma}{m_\pi} = 10 \quad (\ell\sigma m) \]

- NJL and linear sigma models give identical results.
- Explanation: condensate contribution dominates!
- Do the models stand comparison with lattice?

Andersen, TB, PRD 81 (2010) 096004
Thermodynamics of BEC transition

\[ m_\sigma / m_\pi = 9 \quad (\ell_\sigma \text{m}) \]

- NJL and linear sigma models give identical results.
- Explanation: condensate contribution dominates!
- Do the models stand comparison with lattice?

Andersen, TB, PRD 81 (2010) 096004
Simulations at nonzero temperature and diquark source.

Data for pressure and density can be reasonably explained using LO $\chi$PT with source term: dilute Bose gas.
Thermodynamics of BEC transition III

High peak in the energy density!

- Energy dominated by entropy/thermal component.
- Inclusion of thermal order parameter fluctuations needed, NLO $\chi$PT or beyond-mean-field NJL.
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Basic ingredients

Dimensional reduction

4d (Euclidean) quantum field theory at high temperature reduces to a 3d theory of the zero Matsubara mode.

- Heavy modes: “hard mass” $\omega_n = 2\pi n T, n \neq 0$.
- Light modes: “soft mass” $\propto gT$ by loop corrections.
- Dimensionally reduced theory of QCD: EQCD.
- Degrees of freedom: 3d gauge field $A_a +$ adjoint scalar $A_0^a$.

$$\mathcal{L}_{\text{EQCD}} = \frac{1}{4}(F_{ij}^a)^2 + \frac{1}{2}(D_iA_0^a)^2 + \frac{1}{2}m_E^2(A_0^a)^2 + \frac{1}{8}\lambda_E(A_0^aA_0^a)^2$$

- The EFT determines physics on length scales $\propto 1/gT$.

Center symmetry

Global $Z_N$ symmetry of the Yang–Mills theory; its spontaneous breaking is associated with deconfinement phase transition.

- Second/first order transition for two/three colors.
- Order parameter: Polyakov loop.

$$\Omega(x) = \text{Tr} \left\{ \mathcal{P} \exp \left[ ig \int_0^\beta d\tau A_0(\tau, x) \right] \right\}$$

- EQCD breaks $Z_N$ explicitly by expanding around one of the $N$ degenerate minima.

Vuorinen, Yaffe, PRD 74 (2006) 025011

de Forcrand, Kurkela, Vuorinen, PRD 77 (2008) 125014

Zhang, TB, Vuorinen, in progress
Basic degree of freedom: coarse-grained Polyakov loop $\mathcal{Z}$.

$\mathcal{Z}$ acts as an adjoint scalar.

Center symmetry transformation: $\mathcal{Z} \rightarrow \pm \mathcal{Z}$.

$\mathcal{Z}$ is unitary up to a real scale factor:

$$\mathcal{Z} = \frac{1}{2}(\Sigma + i\sigma_a \Pi_a)$$

Superrenormalizable 3d gauge theory of $\mathcal{Z}$:

$$\mathcal{L} = \frac{1}{g_3^2} \left\{ \frac{1}{2} \text{Tr} F_{ij}^2 + \text{Tr} \left( \mathcal{D}_i \mathcal{Z}^\dagger \mathcal{D}_i \mathcal{Z} \right) + V(\mathcal{Z}) \right\}$$

$$V(\mathcal{Z}) = b_1 \Sigma^2 + b_2 \Pi_a^2 + c_1 \Sigma^4 + c_2 (\Pi_a^2)^2 + c_3 \Sigma^2 \Pi_a^2 + d_1 \Sigma^3 + d_2 \Sigma \Pi_a^2$$
## Scales and degrees of freedom

### Scales
- **Scale $T$:** “amplitude mode” $|\mathcal{Z}(x)|$; to be integrated out.
- **Scale $gT$:** electric gluons; phases of $\mathcal{Z}(x)$.
- **Scale $g^2T$:** magnetic gluons; 3d gauge potential $A_i(x)$.

### Couplings
- How to ensure the hierarchy of scales:
  - use global “SU(2)$_L \times SU(2)$_R” symmetry.
  - Preserved by “hard” couplings $h_i$ of order $T$.
  - Broken to SU(2)$_V \times Z_2$ by “soft couplings” $s_i$.

\[
\begin{align*}
    b_1 &= \frac{1}{2} h_1, \\
    b_2 &= \frac{1}{2} (h_1 + g^2 s_1), \\
    d_1 &= \frac{1}{2} g^2 s_4, \\
    d_2 &= \frac{1}{2} g^2 s_5 \\
    c_1 &= \frac{1}{4} h_2 + g^2 s_3, \\
    c_2 &= \frac{1}{4} (h_2 + g^2 s_2), \\
    c_3 &= \frac{1}{2} h_2
\end{align*}
\]
Perturbative matching

- Match to $Z_2$-symmetric one-loop Weiss potential of QCD.
- Reduces to Taylor coefficients (EQCD) and period.
- Domain wall tension predicted at 8% from YM value.

- Explicit $Z_2$ breaking by quarks: use bubble solution.
- Remaining parameter(s) (to be) fixed nonperturbatively.
- Predictions for QC$_2$D thermodynamics as a function of $N_f$, quark masses and chemical potentials (in progress).
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Outlook & challenges

If you are interested in (deconfinement in) cold dense matter, understand available lattice data on dense two-color QCD first!

Hands, Kenny, Kim, Skullerud, 1101.4961 [hep-lat]

Ideal playground for understanding dense matter:
- Simplified modeling due to two-body physics for baryons.
- (Some) lattice data available and more to come.