Physics in pA collisions:
Opportunities and Challenges in Saturation Physics

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Introduction

One-loop Calculation for Forward Hadron Productions in $pA$ Collisions

Gluon Distributions and LO dijet productions

One-loop Calculation for Dijet and Higgs Productions

Conclusion
• **Gluon splitting functions** (胶子劈裂函数) have \(1/z\) singularities →
  Splitting of small-\(x\) gluon is favored → **Gluon density rises** at low \(x \equiv \frac{Q^2}{s}\).
High energy hadrons are composed of a large number of quarks and gluons. When the energy of the interaction is increased, the density of gluons increases. At high energy, the quarks and gluons in the hadron become saturated and begin to experience overlap and recombination. This overlap and recombination leads to non-linear dynamics, which can be described by the BFKL equation.

To separate the saturated dense regime $x < 10^{-2}$ from the dilute regime, we introduce the saturation momentum $Q_s(x)$. This momentum helps to understand the behavior of the hadron at high energy and is crucial for understanding the dynamics of quark-gluon plasma in high energy collisions.
Dual Descriptions of Deep Inelastic Scattering

**Bjorken frame**

\[
F_2(x, Q^2) = \sum_q e_q^2 x \left[ f_q(x, Q^2) + f_{\bar{q}}(x, Q^2) \right].
\]

**Dipole frame** [A. Mueller, 01; Parton Saturation-An Overview]

\[
F_2(x, Q^2) = \sum_f e_f^2 \frac{Q^2}{4\pi^2 \alpha_{em}} \int_0^1 dz \int d^2x_\perp d^2y_\perp \left[ |\psi_T(z, r_\perp, Q)|^2 + |\psi_L(z, r_\perp, Q)|^2 \right] \\
\times \left[ 1 - S^{(2)}(x_\perp, y_\perp) \right], \quad \text{with} \quad r_\perp = x_\perp - y_\perp.
\]

- **Bjorken**: the partonic picture of a hadron is manifest. Saturation shows up as a limit on the occupation number of quarks and gluons.
- **Dipole**: the partonic picture is no longer manifest. Saturation appears as the unitarity limit for scattering. Convenient to resum the multiple gluon interactions.
Saturation physics describes the high density parton distributions in the high energy limit.

- Unitarity of S-matrix $\Leftrightarrow$ Gluon Saturation. (散射矩阵的幺正性$\Leftrightarrow$胶子饱和现象)
- Scatterings involving heavy ions reach saturation faster than proton scatterings.
- Gluon Saturation itself is a very interesting QCD phenomenon, also it provides important information for the initial condition of heavy ion collisions.
- How to identify the smoking guns? 寻找胶子饱和现象的确凿证据！
Recent pA run at the LHC and the ALICE Data

- Unprecedented opportunities to study saturation physics. A lot of rich physics for us to explore both experimentally and theoretically.
- Most saturation models employ LO formula or only part of NLO calculation. Large uncertainty!
- Serious challenges: Factorization issue and NLO correction!
Challenges

$k_t$ factorization formula for the single inclusive gluon productions in hadron-hadron collision:

\[
\frac{d\sigma}{d^2p_T dy} = \frac{2\alpha_s}{C_F p_T^2} \times \\
\times \int d^2k_A,T f_A(x_A, k_A,T) f_B(x_B, p_T - k_A,T).
\]

Inability to find justification of a $k_T$-factorization formula by following chains of citations

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Fundamental to much work in small-$x$ QCD is a $k_T$-factorization formula. Normal expectations in theoretical physics are that when such a result is used, citations should be given to where the formula is justified. We demonstrate by examining the chains of citations back from current work that violations of this expectation are widespread, to the extent that following the citation chains, we do not find a proof or other justification of the formula. This shows a substantial deficit in the reproducibility of a phenomenologically important area of research. Since the published formulae differ in normalization, we test them by making a derivation in a simple model that obeys the assumptions that are stated in the literature to be the basis of $k_T$-factorization in the small-$x$ regime. We find that we disagree with two of the standard normalizations.

- Factorization and NLO calculation? Gluon distribution?
- Difficulties: 1. LO, resumming the past and future gauge links;
  2. NLO, removing divergences.
- Alternative solution: the effective hybrid factorization in pA collisions.
Dilute-Dense (Hybrid) factorizations

The effective Dilute-Dense factorization

\[ \begin{aligned}
\text{projectile: } x_1 & \sim \frac{p_\perp}{\sqrt{s}} e^{+y} \sim 1 \quad \text{valence} \\
\text{target: } x_2 & \sim \frac{p_\perp}{\sqrt{s}} e^{-y} \ll 1 \quad \text{gluon}
\end{aligned} \]

- Protons and virtual photons are dilute probes of the dense gluons inside target hadrons.
- For dijet productions in \( pA \) collisions (2 → 2), there is an effective \( k_t \) factorization.

\[
\frac{d\sigma^{pA\rightarrow ggX}}{d^2P_\perp d^2q_\perp dy_1 dy_2} = x_p g(x_p, \mu) x_A g(x_A, q_\perp) \frac{1}{\pi} \frac{d\hat{\sigma}}{d\hat{t}}.
\]

- For dijet processes in \( pp, AA \) collisions, there is no \( k_t \) factorization [Collins, Qiu, 08], [Rogers, Mulders; 10].
Some relevant experiments

- Forward rapidity physics at RHIC and the LHC(LHCf and TOTEM, etc.) 5 TeV
- Cosmic ray events can have incoming particles with energy up to $10^{12}$ GeV.
- Future EIC or LHeC experiments.
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In a physical process, in order to probe the dense nuclear matter precisely, the proper factorization is required.

Factorization is about separation of short distant physics (perturbatively calculable hard factor) from large distant physics (parton distributions and fragmentation functions).

\[ \sigma \sim x f(x) \otimes H \otimes D_h(z) \]

\(x f(x)\) and \(D_h(z)\) should be universal.

NLO calculation always contains various kinds of divergences.
- Some divergences can be absorbed into \(x f(x)\) or \(D_h(z)\) following evolution equations.
- The rest of divergences should be cancelled.

Hard factor

\[ H = H_{LO}^{(0)} + \frac{\alpha_s}{2\pi} H_{NLO}^{(1)} + \cdots \]

should always be finite and free of divergence of any kind.
Forward hadron production in $pA$ collisions

Consider the inclusive production of inclusive forward hadrons in $pA$ collisions, i.e., in the process: [Dumitru, Jalilian-Marian, 02]

\[ \text{projectile: } x_1 \sim \frac{p_{\perp}}{\sqrt{s}} e^{+y} \sim 1 \quad \text{valence} \]
\[ \text{target: } x_2 \sim \frac{p_{\perp}}{\sqrt{s}} e^{-y} \ll 1 \quad \text{gluon} \]

The leading order result for producing a hadron with transverse momentum $p_{\perp}$ at rapidity $y_h$

\[
\frac{d\sigma_{pA \to hX}^{pA \to hX}}{d^2 p_{\perp} dy_h} = \int_\tau^1 \frac{dz}{z^2} \left[ \sum_f x_p q_f(x_p) \mathcal{F}(k_{\perp}) D_{h/q}(z) + x_p g(x_p) \tilde{\mathcal{F}}(k_{\perp}) D_{h/g}(z) \right].
\]

- The origin of the transverse momentum of the forward parton is from multiple scatterings.
The overall picture of the NLO calculation

[G. Chirilli, BX and F. Yuan, Phys. Rev. Lett. 108, 122301 (2012)] Why is NLO important?

- NLO correction opens up a new channel (parton $k_t$ now can come from hard interaction), therefore it is not small.
- Consistency check of the LO factorization formula.
- Three types of divergences.

NLO calculation for the $q \rightarrow q$ channel


- The rapidity divergence ($y = \frac{1}{2} \ln \frac{l^+}{l^-} \rightarrow -\infty$ light-cone) does not cancel between real and virtual graphs for $k_\perp$ dependent observables.
- Subtractions of the divergences via renormalization group equations; BK and DGLAP.
Factorization for single inclusive hadron productions

- Obtain a systematic factorization for the \( p + A \rightarrow H + X \) process by systematically remove all the divergences!

- Gluons in different kinematical region give different divergences. 1. soft, collinear to the target nucleus; 2. collinear to the initial quark; 3. collinear to the final quark.

\[
P_{p}^{-} \simeq 0
\]
\[
k^{+} \simeq 0
\]
\[
P_{A}^{+} \simeq 0
\]

- All the rapidity divergence is absorbed into the UGD \( F(k_{\perp}) \) while collinear divergences are either factorized into collinear parton distributions or fragmentation functions.

- Consistent check: take the dilute limit, \( k_{\perp}^{2} \gg Q_{s}^{2} \), the result is consistent with the leading order collinear factorization formula.
Numerical implementation of the NLO result

Consistent implementation should include all the $\alpha_s$ (NLL) corrections.

- **NLO parton distributions.** (Choose your favorite one, CTEQ or MSTW)
- **NLO fragmentation function.** (DSS or other FFs.)
- Use NLO hard factors.
- Use the one-loop approximation for the running coupling which is sufficient in this calculation.
- rcBK evolution equation for the dipole gluon distribution.
- Looking at about 30 $-$ 50 percent **uncertainty**. Large $N_c$ limit gives about 10 percent.
- Large NLO correction!
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The unintegrated quark distribution

\[
    f_q(x, k_\perp) = \int \frac{dξ^- d^2ξ_\perp}{4\pi(2\pi)^2} e^{ix^+ξ^- + iξ_\perp \cdot k_\perp} \langle P | \bar{ψ}(0) \mathcal{L}^+ \xi^- \mathcal{L}(ξ^-, ξ_\perp) ψ(ξ_\perp, ξ^-) | P \rangle
\]

as compared to the integrated quark distribution

\[
    f_q(x) = \int \frac{dξ^-}{4\pi} e^{ix^+ξ^-} \langle P | \bar{ψ}(0) \gamma^+ \mathcal{L}(ξ^-) ψ(0, ξ^-) | P \rangle
\]

- The dependence of \( ξ_\perp \) in the definition.
- Gauge invariant definition.
- Light-cone gauge together with proper boundary condition ⇒ parton density interpretation.
- The gauge links come from the resummation of multiple gluon interactions.
- Gauge links may vary among different processes.
For many years, we have know that there are two different gluon distributions:

I. **Weizsäcker Williams** gluon distribution [Kovchegov, Mueller, 98]

$$ xG^{(1)} $$

II. **Color Dipole** gluon distribution [known since early 90s]

$$ xG^{(2)} $$

- **Puzzle:** Why there are two gluon distributions?
  How to distinguish them in experiment? ⋅⋅⋅

  - Quadrupole ⇒ **Weizsäcker Williams** gluon distribution;
  - Dipole ⇒ **Color Dipole** gluon distribution.
A Tale of Twin Gluon Distributions—-绝代双“胶”


Gauge Invariant definitions (规范不变定义): I. Weizsäcker Williams gluon distribution:

\[ xG^{(1)} = 2 \int \frac{d\xi^- d\xi_\perp}{(2\pi)^3 P^+} e^{ixP^+ \xi^- - ik_\perp \cdot \xi_\perp} \text{Tr}\langle P|F^{+i}(\xi^-, \xi_\perp)U^{[+]\dagger}F^{+i}(0)U^{[+]}|P\rangle. \]

II. Color Dipole gluon distributions:

\[ xG^{(2)} = 2 \int \frac{d\xi^- d\xi_\perp}{(2\pi)^3 P^+} e^{ixP^+ \xi^- - ik_\perp \cdot \xi_\perp} \text{Tr}\langle P|F^{+i}(\xi^-, \xi_\perp)U^{[-]\dagger}F^{+i}(0)U^{[+]}|P\rangle. \]

Remarks:
- All physical observable in QCD must be gauge invariant. Both are observables.
- The WW has probabilistic interpretation. \( \Rightarrow \) gluon density (胶子的密度分布).
- Both are fundamental gluon distributions. 绝代双“胶”
A Tale of Twin Gluon Distributions—-绝代双 “胶”

I. Weizsäcker Williams gluon distribution:

\[ xG^{(1)} = 2 \int \frac{d\xi^- d\xi_\perp}{(2\pi)^3} P^+ e^{ixP^+ \xi^- - ik_\perp \cdot \xi_\perp} \text{Tr}(P|F^{+i}(\xi^-, \xi_\perp) U^{[+]\dagger} F^{+i}(0) U^{[+]}) |P\rangle. \]

II. Color Dipole gluon distributions:

\[ xG^{(2)} = 2 \int \frac{d\xi^- d\xi_\perp}{(2\pi)^3} P^+ e^{ixP^+ \xi^- - ik_\perp \cdot \xi_\perp} \text{Tr}(P|F^{+i}(\xi^-, \xi_\perp) U^{[+]\dagger} F^{+i}(0) U^{[+]}) |P\rangle. \]

Questions:

- Can we distinguish these two gluon distributions experimentally?
- How to measure \( xG^{(1)} \) directly? DIS dijet. Important measurement EIC and LHeC.
- How to measure \( xG^{(2)} \) directly? Direct \( \gamma+\text{Jet} \) in \( pA \) collisions and inclusive processes.
In the dijet correlation limit, where \( u = x_1 - x_2 \ll v = z x_1 + (1 - z) x_2 \)

\[ S_{xg}^{(4)}(x_1, x_2; x'_2, x'_1) = \frac{1}{N_c} \left\langle \text{Tr} U(x_1) U^\dagger(x'_1) U(x_2) U^\dagger(x_2) \right\rangle_{xg} \neq S_{xg}^{(2)}(x_1, x_2) S_{xg}^{(2)}(x'_2, x'_1) \]

- Quadrupoles are generically different objects and only appear in dijet processes.
The dijet production in DIS.

Remarks:

- Dijet in DIS is the unique physical process which can measure Weizsäcker Williams gluon distributions.
- Golden measurement for the Weizsäcker Williams gluon distributions of nuclei at small-\(x\). The cross section is directly related to the WW gluon distribution.
- EIC(美国) and LHeC(欧洲核子中心) will provide us perfect machines to study the strong gluon fields in nuclei. Important part in EIC and LHeC physics design.

Di-Hadron correlations in DIS——深度非弹性散射中的双π介子产生

Di-pion correlations at EIC

\[ C(\Delta \phi) = \frac{\int |p_1\perp, |p_2\perp|}{\int |p_1\perp|} \frac{d\sigma^{eA \rightarrow h_1 h_2}}{d\phi_1 dy_1 d^2 p_1 \perp d^2 p_2 \perp} \frac{d\sigma^{eA \rightarrow h_1}}{d\phi_1 dy_1 d^2 p_1 \perp} \]

[Graphs showing di-pion correlations]

[E. Aschenauer, J. H. Lee and L. Zheng, BX, for EIC white paper]

- EIC stage II energy 30 × 100 GeV.
- \( C(\Delta \phi) \) 能够反映双喷注产生的概率
- Physical picture: 核内稠密的胶子会压低双喷注的产生。
Lepton-pair-Hadron correlations in dAu collisions at RHIC

[A. Stasto, BX, D. Zaslavsky, 12]

- $M = 0.5\text{GeV}, 4\text{GeV}$ and $Y = 2.5$.
- Direct measurement of the dipole gluon distribution.
There is no sign of suppression in the $p + p$ and $d + Au$ peripheral data.

The suppression and broadening of the away side jet in $d + Au$ central collisions is due to the multiple interactions between partons and dense nuclear matter (CGC).

Probably the best evidence for saturation. Smoking Gun?
Dijet processes in the large $N_c$ limit in pA collisions

The Fierz identity:

\[ \frac{1}{2} - \frac{1}{2N_c} \quad \text{and} \quad \frac{1}{2} - \frac{1}{2N_c} \]

Graphical representation of dijet processes

- **$g \to q\bar{q}$**: 
  \[ g \to q\bar{q}: \]

- **$q \to qg$**: 
  \[ q \to qg \]

- **$g \to gg$**: 
  \[ g \to gg \]

The Octupole and the Sextupole are suppressed.
Comparing to STAR and PHENIX data measured in $dAu$ collisions

Physics predicted by C. Marquet. Further calculated in [A. Stasto, BX, F. Yuan, 11]

For away side peak in both peripheral (less dense) and central (more saturation) $dAu$ collisions

$$C(\Delta \phi) = \frac{\int |p_1 \perp|, |p_2 \perp| \frac{d\sigma^{pA\rightarrow h_1 h_2}}{dy_1 dy_2 d^2 p_1 \perp d^2 p_2 \perp}}{\int |p_1 \perp| \frac{d\sigma^{pA\rightarrow h_1}}{dy_1 d^2 p_1 \perp}}$$

$$J_{dA} = \frac{1}{\langle N_{coll} \rangle} \frac{\sigma_{pp}^{\text{pair}}}{\sigma_{pp}^{\text{pair}}} \sqrt{\frac{\sigma_{dA}^{\text{pair}}}{\sigma_{dA}^{\text{pair}}}}$$

![Graph showing comparison between peripheral and central $dAu$ collisions](image)
Introduction

One-loop Calculation for Forward Hadron Productions in $pA$ Collisions

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Motivation for Higgs production in pA collisions

Consider the forward Higgs productions in pA collisions: \( p + A \rightarrow H(M_H, k_\perp) + X \)

Comments:

- One-loop calculation allows us to do resummations (Small-x \( \left[ \frac{\alpha_s N_c}{2\pi} \ln \frac{1}{x} \right]^n \) resummation and Sudakov (CSS) \( \left[ \frac{\alpha_s C_F}{2\pi} \ln^2 \frac{Q^2}{Q^2_0} \right]^n \) ) together with multiple scatterings.
- Perform an exact calculation for the leading power contribution for Higgs production.
- The one-loop calculation between Higgs productions and dijet productions are very similar since both \( M_H \gg k_\perp \) and \( M_J \gg k_\perp \).
- This calculation can be generalized to the calculation of the heavy quarkonium \( Y, J/\Psi \) productions.
LO calculation

Gluons couples to the massive scalar through the effective Lagrangian:

\[ \mathcal{L}_{\text{eff}} = -\frac{1}{4} g_\phi \Phi F^{\mu \nu}_\mu F^{\alpha \mu \nu}_\alpha \]

\[ d\sigma^{(\text{LO})} = \sigma_0 \int \frac{d^2 x_\perp d^2 x'_\perp}{(2\pi)^2} e^{ik_\perp \cdot (x_\perp - x'_\perp)} x g_p(x) S_{\gamma \gamma}^{WW}(x_\perp, x'_\perp), \]

- Only initial state interaction is present. ⇒ WW gluon distribution.

\[ S_{\gamma \gamma}^{WW}(x_\perp, y_\perp) = -\left\langle \text{Tr} \left[ \partial_\perp \gamma(x_\perp) U^\dagger(y_\perp) \partial_\perp \gamma(y_\perp) U^\dagger(x_\perp) \right] \right\rangle_Y \]
Some Technical Details


- Power counting analysis: take the leading power contribution in terms of $\frac{k_\perp^2}{Q^2}$.
- Subtraction of the rapidity divergence: $\Rightarrow$ small-$x$ evolution equation

\[ x g_p(x) \int \frac{d\xi}{\xi} K_{\text{DMMX}} \otimes S_{WW}(x_\perp, y_\perp) \]

- Subtraction of the collinear divergence and choose $\mu^2 = \frac{\epsilon_0^2}{r_\perp^2}$:

\[ \left( -\frac{1}{\epsilon} \right) S_{WW}(x_\perp, y_\perp) P_{g/g} \otimes xg(x) \]

- Subtraction of the UV-divergence: $(\alpha_s/\pi) N_c \beta_0 \left( -1/\epsilon_{\text{UV}} + \ln(Q^2/\mu^2) \right)$.
  (Color charge renormalization)
Some Technical Details

Typical diagrams (5 different types in total): $Q^2 = M_H^2$

- Real diagrams ⇒

$$+rac{\alpha_s}{\pi} N_c \left[ \frac{1}{\epsilon^2} - \frac{1}{\epsilon} \ln \frac{Q^2}{\mu^2} + \frac{1}{2} \left( \ln \frac{Q^2}{\mu^2} \right)^2 - \frac{1}{2} \left( \ln \frac{c_0^2}{Q^2 r_{\perp}^2} \right)^2 - \frac{\pi^2}{12} \right].$$

- Virtual graphs ⇒

$$\frac{\alpha_s}{\pi} N_c \left[ -\frac{1}{\epsilon^2} + \frac{1}{\epsilon} \ln \frac{Q^2}{\mu^2} - \frac{1}{2} \ln^2 \frac{Q^2}{\mu^2} + \frac{\pi^2}{2} + \frac{\pi^2}{12} + \beta_0 \ln \frac{Q^2}{\mu^2} \right]$$
For $pA \rightarrow H(Q, k_\perp) + X$: $S_{\text{sud}}(Q^2, r^2_\perp) = \frac{\alpha_s N_c}{\pi} \left[ \frac{1}{2} \ln^2 \frac{Q^2 r^2_\perp}{c^2} - \beta_0 \ln \frac{Q^2 r^2_\perp}{c^2} \right]$

Mismatch between rapidity and collinear divergence between graphs $\Rightarrow S_{\text{sud}}(Q^2, r^2_\perp)$.

Higher loop calculation $\Rightarrow$ exponentiation of $S_{\text{sud}}(Q^2, r^2_\perp)$. 

\[
\frac{d\sigma^{(\text{resum})}}{dy d^2k_\perp} \big|_{k_\perp \ll Q} = \sigma_0 \int \frac{d^2x_\perp d^2x'_\perp}{(2\pi)^2} e^{i k_\perp \cdot r_\perp} e^{-S_{\text{sud}}(Q^2, r^2_\perp)} S_{Y=\ln 1/x_g}(x_\perp, x'_\perp) \times x g_p(x, \mu^2 = c_0^2/r^2_\perp) \left[ 1 + \frac{\alpha_s}{\pi} \frac{\pi^2}{2 N_c} \right].
\]
Sudakov factor for dijet productions in pA collisions and DIS

Consider the dijet productions in pA collisions: [A. H. Mueller, BX, F. Yuan, in preparation.]

\[ k_{\perp} \gg P_{\perp} \]

\[ C_{q\bar{q}} = \frac{N_c}{4} \]
\[ C_{q\gamma} = \frac{N_c}{4} + \frac{N_c}{2} \]
\[ C_{g\rightarrow q\bar{q}} = N_c \]

Comments:

- Two scale problem: \( Q_1^2 = M_H^2 \) and \( Q_2^2 \approx k_{\perp}^2 \)

\[ Q_1^2 \gg Q_2^2 \Rightarrow \frac{\alpha_s C}{2\pi} \ln^2 \frac{Q_1^2}{Q_2^2} \]

- For back-to-back dijet processes, replace \( M_H^2(Q_1^2) \) by \( M_J^2 \approx P_{\perp}^2 \).

- What is different for different channels are just the color factor \( C \).

- Empirical formula for \( C \): \( C = \sum_i \frac{C_i}{2} \), where \( C_i \) is the color factor of the incoming particles. \( C_i = C_F \) for incoming quarks, \( C_i = N_c \) for gluons.

- Obtain complete agreement with the collinear factorization calculation.

- Competition between Sudakov and Saturation suppressions.
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总结:

- **NLO inclusive forward hadron productions** in $pA$ collisions at LHC. *(More precise)*
- **Two fundamental gluon distributions.** 绝代双“胶”
- **DIS dijet provides direct information** of the WW gluon distributions.
- **Important study** at EIC and LHeC.
- **One-loop Calculation for dijet processes (Sudakov Factors) ⇒ The quest for more precise test of the saturation phenomena.** *(A. H. Mueller, BX, F. Yuan, in preparation.)*
- A lot of interesting physics especially in the era of LHC and future EIC.